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The Association
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Draughtsmen.

FUNDAMENTALS OF GEARING.

By THOS. C. F. STOTT, M.I.A.E., A.M.I.Mech.E.

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INTRODUCTION.

This book is intended primarily for the designer and engineer, but will, no doubt, prove equally useful to the student of gearing and gearing problems. In view of the almost universal use of the involute profile for gear teeth, other tooth forms have not been dealt with, and a fair measure of space has been devoted to the mathematics of the involute, in view of its importance in the correct design of high-duty gear teeth.

Spiral bevels and worm gears have not been included, as space will not permit of their full treatment.

I.—THE INVOLUTE.

The involute curve is the path traced out by a point on a cord which is unwound from a drum, the diameter of which is known as the base circle diameter.

Reference to Fig. 1 will make clear certain fundamentals of the involute curve which are used in the design and computation of the involute gear.

(1) The instant arc of any point on an involute curve is normal to a line tangential to the base circle from that point.

(2) The radius of the instant arc at A on an involute curve *a*A will be equal to the straight line AD, *i.e.*, the instant arc at A, has D as its centre.

(3) By the definition of an involute curve, the straight line AD is equal in length to the arc on the base circle *a*D.

(4) Similarly AB = arc *ab*
BC = arc *bc*
i.e., the distance, measured on a straight line tangential to the base circle (normal to the involute curves) between any two involute curves generated from the same base circle, is equal to the arc on the base circle subtended by their origins.

When the involute curve is used for a tooth profile, the origin of the involute is determined by the pressure angle required at a certain radius. The pressure angle at a point on an involute curve is the angle between a tangent to the curve at that point and a line from that point to the centre of the base circle. It will be

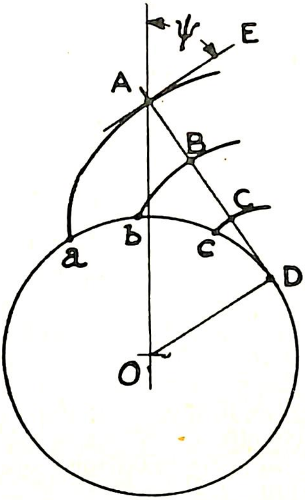


Fig. 1.

seen that the pressure angle of an involute curve is not constant, but varies, depending on the radius at which it is taken. When the pressure angle of a gear tooth is given, it is understood to be the pressure angle at the pitch radius, unless otherwise stated.

Referring to Fig. 1, ψ is the pressure angle at point A.

$A\hat{D}O$ = rt. angle.
AE is tangent to the involute curve at A.
Therefore AE is at rt. angles to AD.

$$\psi + O\hat{A}D = \text{rt. angle} = O\hat{A}D + D\hat{O}A.$$

Therefore $D\hat{O}A = \psi$ = pressure angle at radius OA.

The pressure angle at any given radius may be determined, knowing the base circle radius. Referring to Fig. 2,

$$\text{Cos } \psi_1 = R_b/r$$

Alternatively, knowing the pressure angle at a given radius, the pressure angle at any other radius may be found.

$$R_b/r = \cos \psi_1 \text{ and } R_b/R = \cos \psi_2$$
$$R_b = r \cos \psi_1 = R \cos \psi_2$$

$$\text{Therefore } \cos \psi_2 = \frac{r \cos \psi_1}{R}$$

where R_b = Base circle radius.
r = Given radius.
R = Radius at which pressure angle is required.
ψ_1 = Pressure angle at radius r.
ψ_2 = Pressure angle at radius R.

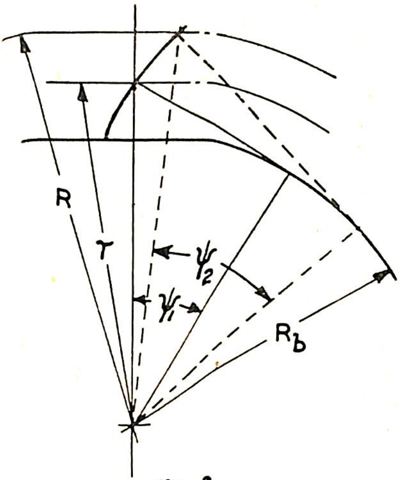

R
r
ψ_2
ψ_1
R_b

Fig. 2.

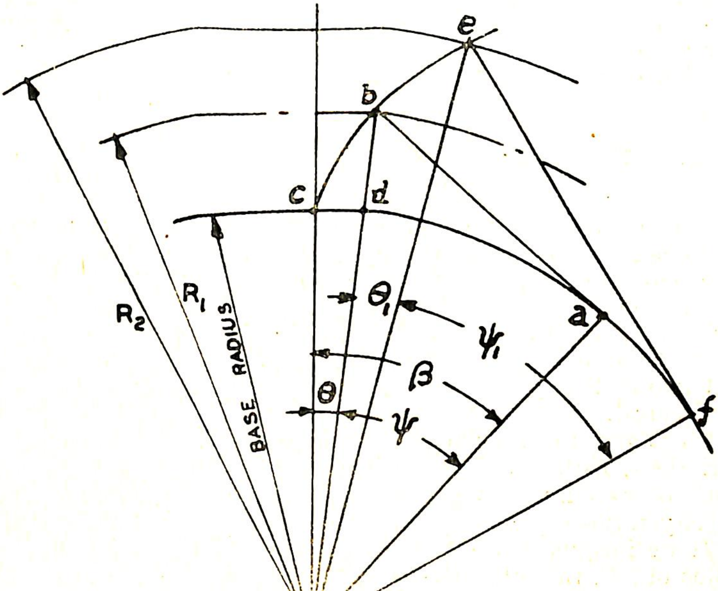

e
b
c
d
a
f
R_2
R_1
BASE RADIUS
θ_1
ψ_1
β
θ
ψ
O

Fig. 3.

The Involute Function.

It will be seen from reference to Fig. 3 that the vectorial angle of an involute curve from its origin is a function of the pressure angle at the radius at which the vectorial angle is taken. (The pressure angle at the point of origin is zero).

The vectorial angle at *b* on the involute curve is θ, which is the angle subtended by *o c* (the radial line through the origin) and *o b*—the pressure angle at *b* at radius R_1 being ψ. The angle θ in radians is known as the involute function of the angle ψ and is written "inv. ψ."

Referring to Fig. 3,

β (radians) $=$ Arc ac/ao.
Tan $\psi = ab/ao$. But $ab =$ arc ac.
Therefore β (radians) $= \tan \psi$.
ψ (radians) $=$ Arc ad/ao.
Therefore θ (radians) $= \beta - \psi$.
$= \tan \psi - \psi$ (radians) $=$ inv. ψ.

Note that the involute function is always the vectorial angle (in radians) taken from the origin of the involute on the base circle.

If it is required to find the vectorial angle θ_1 *i.e.*, the angle subtended by radial lines to points *b* on radius R_1 and *e* on radius R_2 on the involute, this is the difference between the involute function of the pressure angle (ψ_1) at radius R_2 and the involute function of the pressure angle (ψ) at radius R_1.

θ_1 (radians) $=$ inv. $\psi_1 -$ inv. ψ.

II.—THE APPLICATION OF THE INVOLUTE FORM TO GEAR TEETH.

The function of a pair of mating gears is to transmit motion from one shaft to another with uniformity of velocity ratio. Either the cycloidal or the involute form tooth will satisfy this condition, but the cycloidal has the inherent disadvantage that a slight variation of the theoretical centre distance destroys the uniformity of angular velocity. It is very difficult, if not impossible, to ensure that two mating gears run on their exact theoretical centres under all circumstances, and it is possibly this fact more than any other that has put the involute form tooth into almost universal use. Variations, within fairly wide limits, of the theoretical centre distance of two involute gears does not impair the uniformity of velocity ratio—a necessity for quiet running and efficient gears.

If we imagine two drums as indicated in Fig. 4 (one having a radius of AC, the other BD) and a cord run from one to the other (CD), winding of the cord on one drum will unwind it from the other and uniformity of velocity ratio will result between the two drums. The velocity ratio will be the ratio of the diameters of the two drums.

To apply this principle to the involute gear, the conditions of running must be such that contact between the teeth always occurs on the line tangential to the base circles. This line is known as the "line of action."

Referring to Fig. 4, let *a* be a point on one involute tooth contacting with point *b* on the other involute tooth. The instant arc of any point on an involute curve is normal to a line tangential to the base circle, from that point.

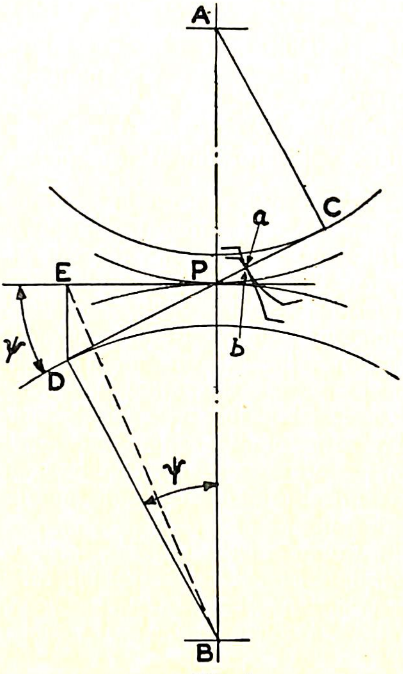


Fig. 4.

Also when two curves touch, they have a common normal. Hence the common normal to the points *a* and *b* must also be the common tangent to the base circles whose radii are AC and BD. Therefore, with teeth of involute profile, the point of contact is always on the line of action DC.

Comparing the similar triangles ACP and BDP—

$$BD : AC :: BP : AP$$

thus the common normal to the teeth or line of action always passes through the pitch point P. The pitch point may be said to be the point at which the line of action intersects the line joining the two gear centres. With a pair of mating gears running on their correct theoretical centres, the pitch point is also the point at which the pitch diameters touch. When the involute curve is applied to gear tooth form, the pressure angle of the tooth is the pressure angle taken at the pitch diameter.

Referring again to Fig. 4,

Angles $P\hat{E}D$, $B\hat{P}E$ and $P\hat{D}B$ are right angles.

$$D\hat{P}E + B\hat{P}D = \text{rt. angle.}$$

$$D\hat{B}P + B\hat{P}D = \text{rt. angle.}$$

Therefore $D\hat{P}E = D\hat{B}P = \psi =$ pressure angle at the pitch point.

Since $DB/BP = \cos \psi$
$DB = BP \cos \psi$
i.e., Base circle radius = Pitch radius × Cos pressure angle.

Since the instant arc of any point on an involute curve is normal to a line tangential to the base circle from that point, and when two curves touch they have a common normal, if the centres are closed together or moved apart, contact still occurs on the line of action tangential to the two base circles, but the angle made by this new line of action and a normal to the perpendicular line joining the two gear centres, *i.e.*, the pressure angle at the pitch point, is either decreased or increased, depending on whether the centres are decreased or increased. The base circle radii remaining unaltered, the velocity ratio remains unaltered, and as contact still occurs on the line of action tangential to the base circles, uniformity of velocity ratio is still obtained, although the theoretical centres have been departed from.

Another point in favour of the involute form tooth is that the mating rack has straight sides. Gears generated by the hobbing process and rack planing methods use cutters of the basic rack form which, being straight-sided, can be both accurately and cheaply produced.

Referring to Fig. 5, contact between two mating teeth occurs on the line of action PA. Also, as the instant arc of any point on an involute curve is normal to a line tangential to the base circle from that point, it follows that if contact between the rack and gear is taking place, the instant arc of the rack at the point of contact must also be normal to a line tangential to the base circle from that point. The rack being a gear of an infinite diameter, the base circle must also be of an infinite diameter, and again, the instant radius of the involute curve must also be of an infinite length, *i.e.*, a straight line.

Furthermore, the rack side at points P and C is normal to the line of action, and as the line of action is a straight line, the angle of the rack side must also be a straight line normal to the line of action at the point of contact.

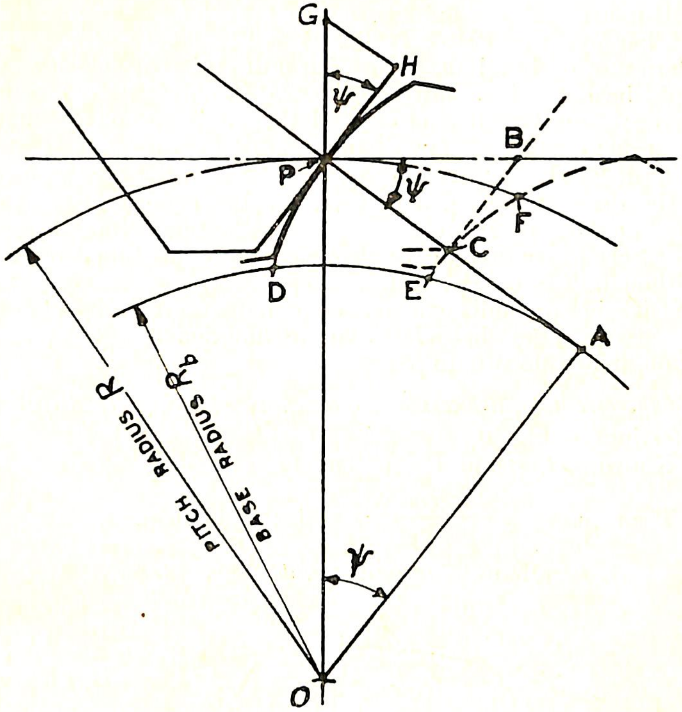


Fig. 5.

From the similar triangles POA and GPH it will be seen that angle $P\hat{O}A$ = angle $G\hat{P}H$. Therefore the angle of the side of the rack is equal to the pressure angle of the gear on the pitch line. (When speaking merely of the pressure angle of a gear tooth, it is understood to mean the pressure angle on the pitch radius).

If it is assumed that the rack has moved from P to B, then for uniformity of velocity ratio, PB on the pitch line of the rack must equal arc PF on the pitch line of the gear.

Distance moved along line of action = PC.

It is evident that angle $B\hat{P}C = \psi$.

Therefore PB = PC/Cos ψ.

Also distance PC on the line of action is equal to the arc DE on the base circle (origin of involutes on same base circle).

$$\text{Pitch circle circumference} = \frac{\text{Base circle circumference}}{\text{Cos } \psi}$$

Therefore for same angular movement,

Arc PF = Arc DE/Cos ψ.

But arc DE = Line PC,

Therefore arc PF = PC/Cos ψ = PB.

Therefore motion is taking place with uniformity of velocity ratio. It should be borne in mind that the two pitch circles of a pair of mating gears or the pitch circle and the pitch line of a gear and rack, roll one on the other without slip.

While it is possible to solve many gearing problems graphically or on the drawing board, a mathematical solution is often to be preferred or required as a check. In the solution of practically all problems relating to involute teeth, the involute function is required, and although this can be obtained easily for any angle, a table is given in the appendix which will perhaps serve its most useful purpose as a guide when the involute function is given and the corresponding angle is required.

To find the circular thickness of tooth at any radius (external gear).

Referring to Fig. 6.

Knowing t, r and R, to find T.

Then $\cos \beta = \dfrac{r \cos \psi}{R}$ Find β in degrees.

$$\theta \text{ radians} = t/2r + \text{inv. } \psi - \text{inv. } \beta.$$

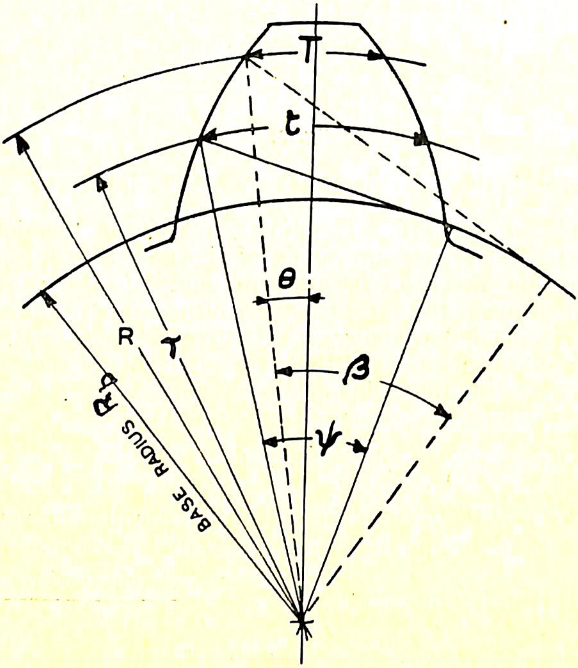


Fig. 6.

Therefore $T = 2R\theta = 2R(t/2r + \text{inv. } \psi - \text{inv. } \beta)$
where t = Circular thickness at radius r.
T = Circular thickness at radius R.
r = Known radius.
R = Radius at which circular thickness is required
ψ = Pressure angle at radius r.
β = Pressure angle at radius R.

To find circular thickness of tooth at any radius (internal gear).

Referring to Fig. 7.

Knowing t, r, and R, to find T.

Then $\cos \beta = \dfrac{r \cos \psi}{R}$ Find β in degrees.

θ radians $= t/2r + \text{inv. } \beta - \text{inv. } \psi$.
Therefore $T = 2R\theta$ radians.
$= 2R(t/2r + \text{inv. } \beta - \text{inv. } \psi)$
where t = Circular thickness at radius r.
T = Circular thickness at radius R.
r = Known radius.
R = Radius at which circular thickness is required.
ψ = Pressure angle at radius r.
β = Pressure angle at radius R.

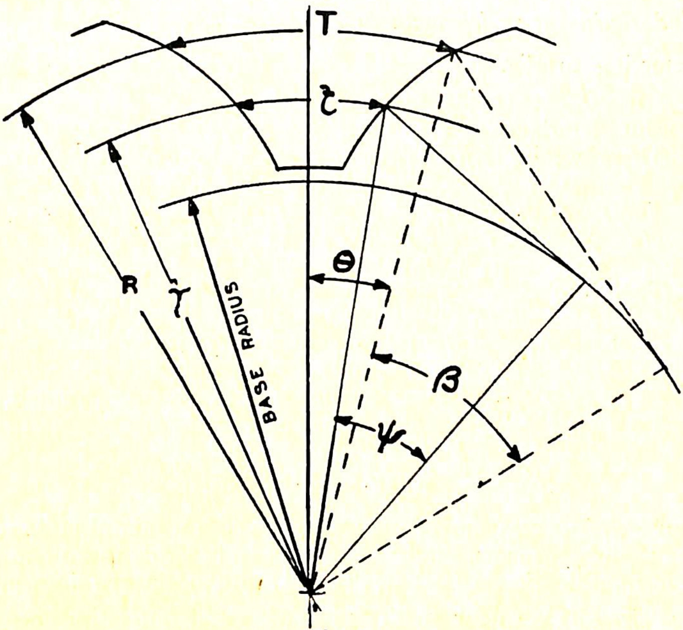


Fig. 7.

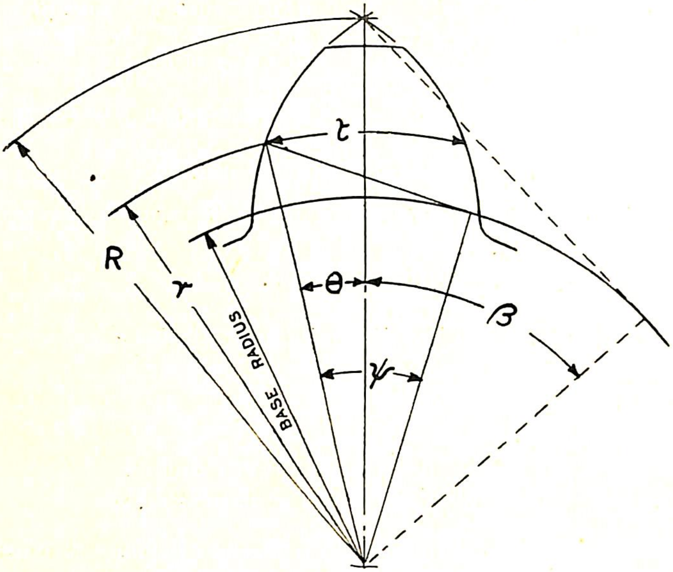


Fig. 8.

To find radius at which tooth becomes pointed.

Referring to Fig. 8.

Inv. $\beta = \theta$ radians + inv. ψ.
But θ radians $= t/2r$.
Therefore inv. $\beta = t/2r +$ inv. ψ. Find β in degrees.

$$\text{Then } \cos\beta = \frac{r\cos\psi}{R}$$

$$\text{Therefore } R = \frac{r\cos\psi}{\text{Cos }\beta}$$

where t = Circular thickness at radius r.
ψ = Pressure angle at radius r.
R = Radius at which tooth becomes pointed.
β = Pressure angle at radius R.

III.—STRAIGHT SPUR GEARS.

Spur gearing may be taken as the basic and simplest form of gearing; other forms, such as helical, spiral and bevel, are in reality a modification of the spur gear to suit special requirements.

The size of the gear teeth is usually designated by the diametral pitch in the inch system of measurement and by the metric modula in the metric system. The English modula is also sometimes used, the modula in this case being $\frac{1}{\text{Diametral pitch}}$

The diametral pitch of a gear is the number of teeth per inch of pitch diameter. Thus the diametral pitch of a 21-tooth gear having a pitch diameter of 3 inches is 7 D.P. The tooth proportions are based on the diametral pitch or the circular pitch and are given in a table in the appendix for easy reference. It will be noted that the standard clearances are given as $\frac{\cdot 157}{\text{Diametral pitch}}$ but in the Fellows Gear Shaper Company standard, the clearance is increased to $\frac{\cdot 250}{\text{Diametral pitch}}$, *i.e.*, ·25 of the addendum for pitches up to about 16 D.P. A gear cut to Fellows Gear Shaper Company standard, therefore, has a greater dedendum than standard.

From the following formulae, the proportions of the required gear may be calculated. (All dimensions in inches).

Reference Fig. 9.

(1) Diametral pitch $= \frac{\text{Number of teeth}}{\text{Pitch diameter}}$

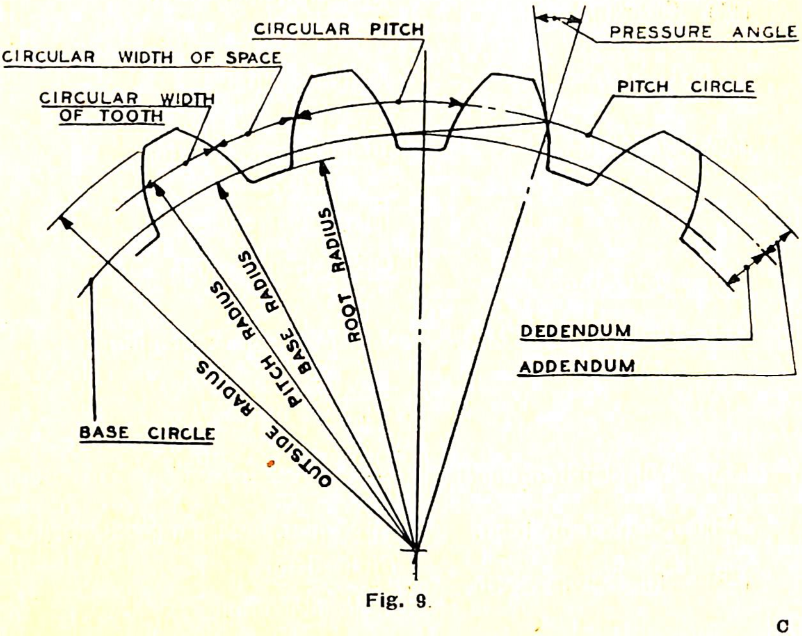


Fig. 9.

(2) Diametral pitch $= \dfrac{\pi}{\text{Circular pitch}}$

(3) Circular pitch $= \dfrac{\pi}{\text{Diametral pitch}}$

(4) Circular pitch $= \dfrac{\text{Pitch diam.} \times \pi}{\text{Number of teeth}}$

(5) Pitch diam. $= \dfrac{\text{Number of teeth}}{\text{Diametral pitch}}$

(6) Pitch diam. $= \dfrac{\text{Circular pitch} \times \text{No. of teeth}}{\pi}$

(7) Addendum $= \dfrac{1}{\text{Diametral pitch}}$

(8) Addendum $= \dfrac{\text{Circular pitch}}{\pi}$

(9) Working depth $= 2 \times \text{Addendum}$

(10) Clearance $= \dfrac{\cdot 157}{\text{Diametral pitch}}$

(11) Clearance $= \dfrac{\cdot 157 \times \text{Circular pitch}}{\pi}$

$= \dfrac{\text{Circular pitch}}{20}$

(12) Dedendum $= \text{Addendum} + \text{Clearance}$

$= \dfrac{1{\cdot}157}{\text{Diametral pitch}}$

(13) Dedendum $= \dfrac{1{\cdot}157 \text{ Circular pitch}}{\pi}$

(14) Full depth of tooth $= \text{Addendum} + \text{Dedendum}$

$= \dfrac{2{\cdot}157}{\text{Diametral pitch}}$

(15) Full depth of tooth $= \dfrac{2{\cdot}157 \text{ Circular pitch}}{\pi}$

(16) Number of teeth $= \text{Pitch diam.} \times \text{Diametral pitch.}$

(17) Number of teeth $= \dfrac{\text{Pitch diam.} \times \pi}{\text{Circular pitch}}$

(18) Circular thickness of tooth $= \dfrac{\text{Circular pitch}}{2}$

(From the above must be deducted the running backlash).

(19) Circular width of tooth space $= \dfrac{\text{Circular pitch}}{2}$

(To the above must be added the running backlash).

(20) Outside diameter $= \dfrac{\text{Number of teeth} + 2}{\text{Diametral pitch}}$

(21) Root diameter $= \dfrac{\text{No. of teeth} - 2{\cdot}314}{\text{Diametral pitch}}$

(22) Base circle diam. = Pitch diam. × Cos pressure angle.

(23) Base pitch = Circular pitch × Cos pres. angle.

(24) Centre distance $= R + r = \dfrac{N + n}{2\,DP}$

where R = Pitch radius of gear.
r = Pitch radius of pinion.
N = Number of teeth in gear.
n = Number of teeth in pinion.
DP = Diametral pitch.

(25) Root diameter (for internal gears) $= \dfrac{\text{No. of teeth} + 2{\cdot}314}{\text{Diametral pitch}}$

(26) Internal diameter (for internal gears) $= \dfrac{\text{No. of teeth} - 2}{\text{Diametral pitch}}$

(27) Centre distance (for internal gears) $= R - r = \dfrac{N - n}{2\,DP}$

where R = Pitch radius of internal gear.
r = Pitch radius of pinion.
N = No. of teeth in internal gear.
n = No. of teeth in pinion.
DP = Diametral pitch.

Stub Teeth.

To avoid undercutting on gears with a small number of teeth and also to obtain a stronger tooth, the stub tooth was introduced. In this system the tooth size is designated by the use of two pitches —thus 8/10. In this case the pitch diam., number of teeth and circular pitch, are based on a standard 8 diametral pitch gear, but the addendum, clearance and dedendum are based on a standard 10 diametral pitch gear.

Tooth Action of Straight Spur Gears.

For continuity of tooth action between two mating spur gears it is essential that before one tooth of a gear comes out of mesh with the mating gear another tooth must enter into engagement. If the second tooth comes into engagement the instant before the first tooth leaves engagement, the full tooth load will come on to the tip of the second tooth. If, however, the first tooth does not leave engagement until contact is well down the second tooth, the tip of the second tooth will only have half the full tooth load imposed upon it.

It will be seen that apart from quietness of running and continuity of tooth action, it is advisable to have as many teeth in mesh as possible to share the load and under the condition of one tooth taking the whole of the load, that load will then be taken some way down the tooth and not on the tip. Considering the tooth as a cantilever, the actual position of the applied load has far-reaching effects on the tooth strength.

The arc of action is the arc on the pitch circle that the tooth travels from the time it enters into contact with the mating tooth to the time contact with the mating tooth ceases.

$$\text{Arc of action (radians)} = \frac{\text{Arc of action}}{\text{Pitch radius}}$$

or Arc of action = Arc of action (radians) × Pitch radius.

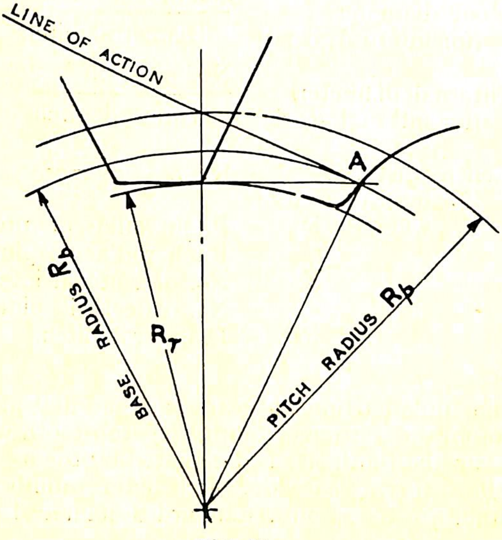


Fig. 10.

The length of the line of contact is the distance along the line of action that the tooth travels from the time it enters into contact with the mating tooth to the time contact with the mating tooth ceases.

Therefore an arc on the base circle diam. equal to the length of the line of contact must be equal to the arc on the base circle diam. subtended by the arc of action (radians).

The arc on the base circle diam. subtended by the arc of action (radians).

$$= \text{Arc of action (radians)} \times \text{Base circle radius.}$$

$$= \text{Arc of action (radians)} \times \text{Pitch radius} \times \text{Cos pressure angle.}$$

$$= \frac{\text{Arc of action}}{\text{Pitch radius}} \times \text{Pitch radius} \times \text{Cos pressure angle.}$$

$$= \text{Arc of action} \times \text{Cos pressure angle.}$$

Therefore,

$$\text{Length of line of contact} = \text{Arc of action} \times \text{Cos pressure angle.}$$

The *Contact Ratio* is the ratio $\dfrac{\text{Arc of action}}{\text{Circular pitch}}$

As length of line of contact = Arc of action × Cos pressure angle, and Base pitch = Circular pitch × Cos pressure angle, the contact ratio may therefore be stated as the ratio,

$$\frac{\text{Length of line of action}}{\text{Base pitch}}$$

in which form it is easier for computation.

The number of pitches in mesh is expressed by the contact ratio. Thus, if the contact ratio is 1·0, one tooth is just leaving engagement as another is just entering, an undesirable condition with spur gears. If with standard gears a contact ratio of from 1·3 to 1·5 is unobtainable, the teeth should be corrected to give the desired ratio.

Referring to Fig. 11,

$Bx = R_b \cos \psi$

$AB =$ Centres of gears. Therefore $Ax = AB - Bx$

$ax = R_b \sin \psi$

Therefore $r = \sqrt{ax^2 + Ax^2}$

As contact can only occur on the line of action, it follows that if the outside radius of gear A is greater than radius r, then a condition as shown in Fig. 14 arises. The top of the tooth shown shaded is useless and will only cause rubbing at the corner of the pinion where the involute ceases. The shaded portion of the tooth, therefore, could be removed, *i.e.*, the tooth "topped" without in any way affecting the tooth action. If, however, the length of the line of contact AB is insufficient, the gear should be corrected.

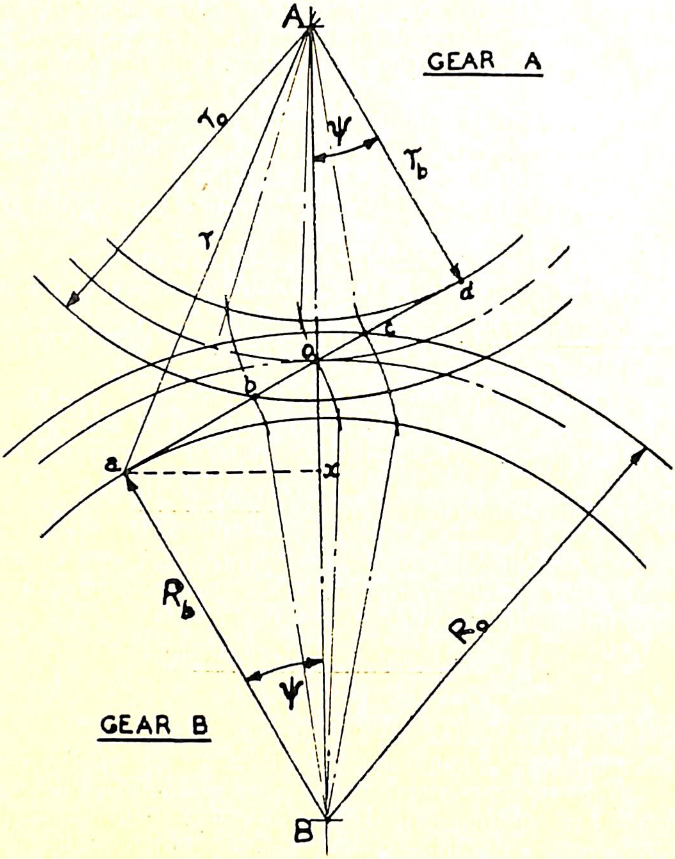


Fig. 11.

For the involute not to be destroyed above the base circle diameter by the tip of the rack form cutter, contact between the cutter and the gear being cut must always be on the line of action. Referring to Fig. 10, the line of action ceases where it contacts the base circle at A, therefore at this point contact must be with the tip of the cutter just going out of action.

R_r is the minimum root radius of the gear for the involute not to be destroyed above the base circle diameter.

$R_r = R_b \cos \psi$

But $R_b = R_p \cos \psi$ Therefore $R_r = R_p \cos^2 \psi$

Dedendum $= R_p - R_p \cos^2 \psi$

$= R_p (1 - \cos^2 \psi)$

Dedendum (Std.) $= 1{\cdot}157/p_d$

Dedendum (Fellows) $= 1{\cdot}25/p_d$

And $R_p = N/2\,p_d$

Therefore,

$$N\,(1-\cos^2\psi)/2\,p_d = 1{\cdot}157/p_d \text{ (Std.) or } 1{\cdot}25/p_d \text{ (Fellows).}$$
$$N\,(1-\cos^2\psi)/2 = 1{\cdot}157 \text{ (Std.) or } 1{\cdot}25 \text{ (Fellows).}$$

Therefore,

$$N = \frac{2{\cdot}314}{1-\cos^2\psi} \text{ (Std.)}$$

$$\text{or } N = \frac{2{\cdot}5}{1-\cos^2\psi} \text{ (Fellows)}$$

N = Minimum number of teeth for involute not to be destroyed above the base circle.

where R_p = Pitch radius.
R_b = Base radius.
R_r = Root radius.
ψ = Pressure angle.
p_d = Diametral pitch.

In a number of cases the tooth action is such that contact between the mating teeth ceases some little way above the base circle diameter. It will be seen that in certain instances the destruction of part of the involute adjacent to the base circle does not necessarily interfere with the tooth action as the part of the involute removed is not used. In any case it is advisable to avoid, if possible, contact on or very near the base circle, as this part of the involute is the most difficult to produce with great accuracy.

To find length of line of contact and contact ratio between two spur gears.

Referring to Fig. 11,

$ac = \sqrt{R_0^2 - R_b^2}$
$ao = R_b \tan\psi$
$oc = ac - ao = \sqrt{R_0^2 - R_b^2} - R_b \tan\psi$
$bd = \sqrt{r_0^2 - r_b^2}$
$od = r_b \tan\psi$
$bo = bd - od = \sqrt{r_0^2 - r_b^2} - r_b \tan\psi$
$bc = bo + oc = \sqrt{r_0^2 - r_b^2} + \sqrt{R_0^2 - R_b^2} - (r_b + R_b)\tan\psi$

Length of line of contact

$$= \sqrt{r_0^2 - r_b^2} + \sqrt{R_0^2 - R_b^2} - (r_b + R_b)\tan\psi$$

$$\text{Contact ratio} = \frac{\text{Length of line of contact}}{\text{Base pitch}}$$

$$= \frac{\sqrt{r_0^2 - r_b^2} + \sqrt{R_0^2 - R_b^2} - (r_b + R_b)\tan\psi}{\text{Circular pitch} \times \text{Cos }\psi}$$

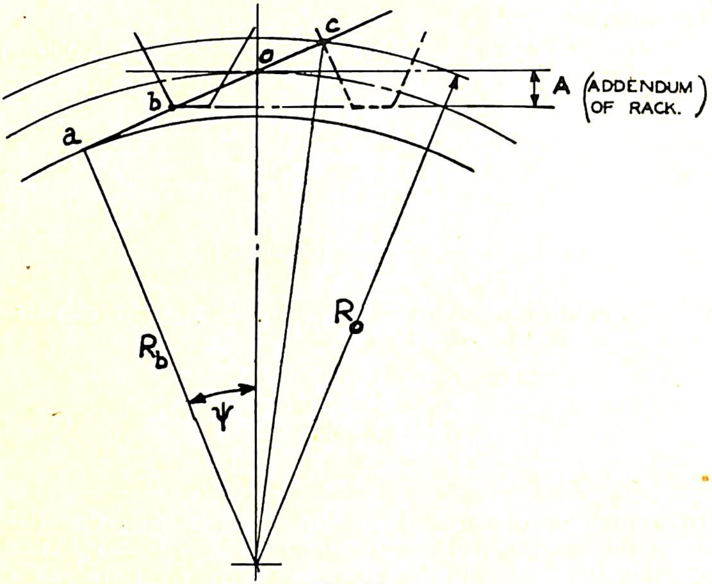


Fig. 12.

where R_0 = Outside rad. of gear.
R_b = Base rad. of gear.
r_0 = Outside rad. of pinion.
r_b = Base rad. of pinion.
ψ = Pressure angle.

To find the length of line of contact and contact ratio of a spur gear and rack.

Referring to Fig. 12,

$$ac = \sqrt{R_0^2 - R_b^2}$$
$$ao = R_b \tan \psi$$

Therefore

$$oc = ac - ao = \sqrt{R_0^2 - R_b^2} - R_b \tan \psi$$
$$bo = A \operatorname{cosec} \psi$$
$$bc = bo + oc$$
$$= A \operatorname{cosec} \psi + \sqrt{R_0^2 - R_b^2} - R_b \tan \psi$$

Length of line of contact

$$= A \operatorname{cosec} \psi + \sqrt{R_0^2 - R_b^2} - R_b \tan \psi$$

$$\text{Contact ratio} = \frac{\text{Length of line of contact}}{\text{Base pitch}}$$

$$= \frac{A \operatorname{cosec} \psi + \sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{\text{Circular pitch} \times \operatorname{Cos} \psi}$$

where R_0 = Outside rad. of gear.
R_b = Base rad. of gear.
A = Addendum of rack.
ψ = Pressure angle.

To find length of line of contact and contact ratio of a spur gear and internal gear.

Referring to Fig. 13,

$$
\begin{aligned}
ac &= \sqrt{R_1^2 - R_b^2} \\
ao &= R_b \tan \psi \\
co &= ao - ac = R_b \tan \psi - \sqrt{R_1^2 - R_b^2} \\
bd &= \sqrt{r_0^2 - r_b^2} \\
bo &= r_b \tan \psi \qquad od = \sqrt{r_0^2 - r_b^2} - r_b \tan \psi \\
cd &= co + od \\
&= R_b \tan \psi - \sqrt{R_1^2 - R_b^2} + \sqrt{r_0^2 - r_b^2} - r_b \tan \psi
\end{aligned}
$$

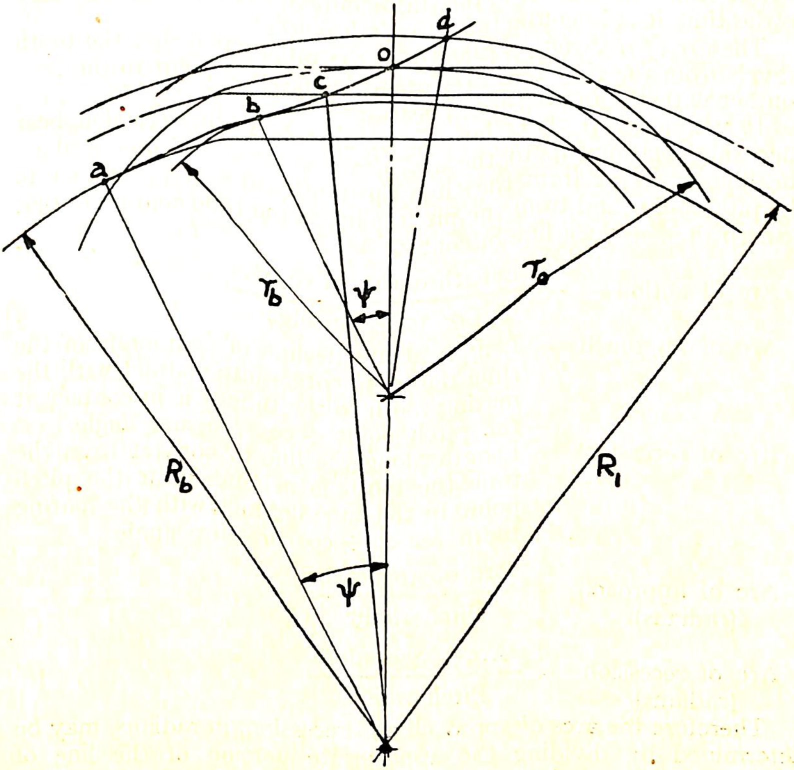


Fig. 13.

Length of line of contact

$$= (R_b - r_b) \tan \psi - \sqrt{R_1^2 - R_b^2} + \sqrt{r_0^2 - r_b^2}$$

$$\text{Contact ratio} = \frac{\text{Length of line of contact}}{\text{Base pitch}}$$

$$= \frac{(R_b - r_b) \tan \psi - \sqrt{R_1^2 - R_b^2} + \sqrt{r_0^2 - r_b^2}}{\text{Circular pitch} \times \operatorname{Cos} \psi}$$

where R_1 = Internal rad. of internal gear.
R_b = Base rad. of internal gear.
r_0 = Outside rad. of spur gear.
r_b = Base rad. of spur gear.
ψ = Pressure angle.

The arc of action is divided into the arc of approach and the arc of recession.

Arc of action = arc of approach + arc of recession.

The *arc of approach* is the arc on the pitch circle that the tooth travels from the time it enters into contact with the mating tooth to the time it is in contact at the pitch point.

The *arc of recession* is the arc on the pitch circle that the tooth travels from the time it is in contact at the pitch point to the time contact with the mating tooth ceases.

It will be seen that the arcs of approach and recession bear the same relationship to the arc of action as the distances along the line of action from the time the tooth enters into contact to the pitch point and from the pitch point to the time contact ceases, bear to the length of line of contact.

$$\text{Arc of action} = \frac{\text{Length of line of contact}}{\text{Cos pressure angle}}$$

Arc of approach = Length along the line of contact from the time the tooth enters into contact with the mating tooth to the time it is in contact at the pitch point ÷ cos pressure angle.

Arc of recession = Length along the line of contact from the time the tooth is in contact at the pitch point to the time contact with the mating tooth ceases ÷ cos pressure angle.

$$\text{Arc of approach (radians)} = \frac{\text{Arc of approach}}{\text{Pitch radius}}$$

$$\text{Arc of recession (radians)} = \frac{\text{Arc of recession}}{\text{Pitch radius}}$$

Therefore the arcs of approach and recession, in radians, may be determined by dividing the appropriate portion of the line of contact by the pitch radius × cos pressure angle (*i.e.*, the base circle radius).

To find arcs of approach and recession of two mating spur gears.

Referring to Fig. 11. Assuming gear A to be the driver.

Arc of approach of gear A $= oc/\cos\psi$

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan\psi}{\text{Cos}\ \psi}$$

Arc of approach of gear A (radians)

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan\psi}{r_b}$$

Arc of recession of gear A $= bo/\cos\psi$

$$= \frac{\sqrt{r_0^2 - r_b^2} - r_b \tan\psi}{\text{Cos}\ \psi}$$

Arc of recession of gear A (radians)

$$= \frac{\sqrt{r_0^2 - r_b^2} - r_b \tan\psi}{r_b}$$

where R_0 = Outside rad. of gear.
R_b = Base rad. of gear.
r_0 = Outside rad. of pinion (gear A).
r_b = Base rad. of pinion (gear A).
ψ = Pressure angle.

To find arcs of approach and recession of a spur gear and rack.

Referring to Fig. 12. Assuming gear to be the driver.

Arc of approach of gear $= bo/\cos\psi$

$$= \frac{A \text{ cosec } \psi}{\text{Cos}\ \psi}$$

Arc of approach of gear (radians)

$$= \frac{A \text{ cosec } \psi}{R_b}$$

Arc of recession of gear $= oc/\cos\psi$

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan\psi}{\text{Cos}\ \psi}$$

Arc of recession of gear (radians)

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan\psi}{R_b}$$

where R_0 = Outside rad. of gear.
R_b = Base rad. of gear.
A = Addendum of rack.
ψ = Pressure angle.

To find arcs of approach and recession of a spur gear and internal gear.

Referring to Fig. 13. Assuming the spur gear to be the driver.

Arc of approach of spur gear $= co/\cos \psi$

$$= \frac{R_b \tan \psi - \sqrt{R_1^2 - R_b^2}}{\operatorname{Cos} \psi}$$

Arc of approach of spur gear (radians)

$$= \frac{R_b \operatorname{Tan} \psi - \sqrt{R_1^2 - R_b^2}}{r_b}$$

Arc of recession of spur gear

$$= od/\cos \psi$$

$$= \frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{\operatorname{Cos} \psi}$$

Arc of recession of spur gear (radians)

$$= \frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{r_b}$$

where R_1 = Internal rad. of internal gear.
R_b = Base rad. of internal gear.
r_0 = Outside rad. of spur gear.
r_b = Base rad. of spur gear.
ψ = Pressure angle.

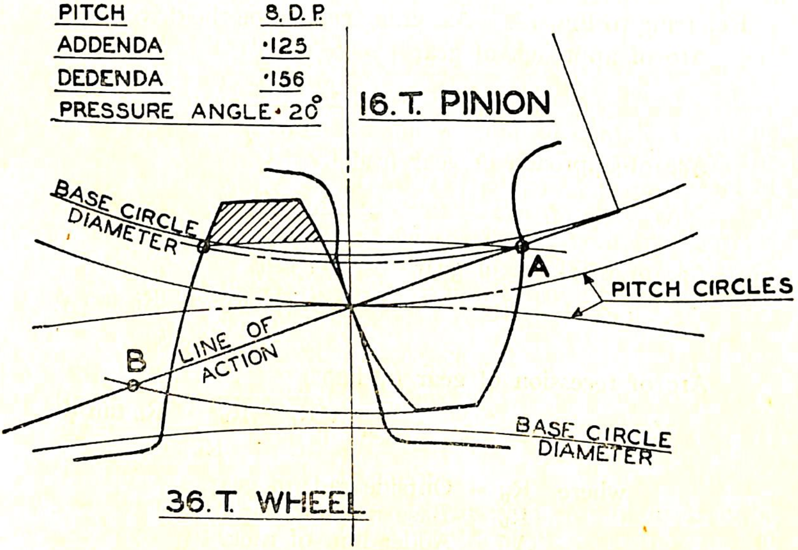
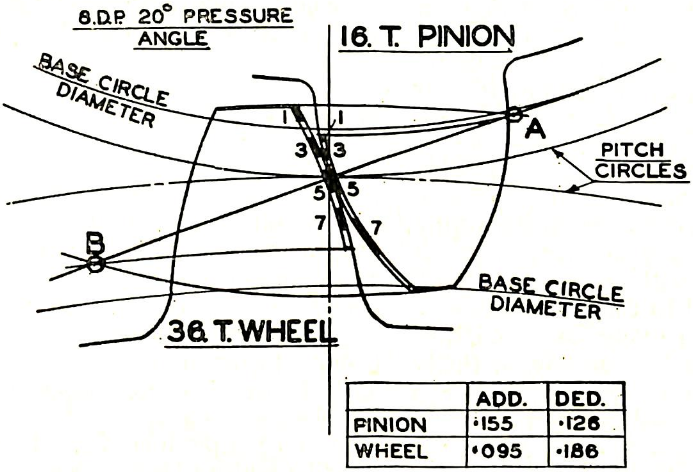


Fig. 14.

Correction of Spur Gear Teeth.

Gears of a small number of teeth cut by the generating process are subjected to undercutting and interference. Undercutting can weaken the tooth considerably and the removal of some of the involute profile above the base circle can seriously affect the tooth action when the gears have a small number of teeth. Fig. 14 illustrates a 16-tooth pinion meshing with a 36-tooth wheel. The tooth proportions in this instance are Fellows Standard. Contact can only occur on the line of action, and as part of the involute profile on the 16-tooth pinion has been destroyed, due to interference, contact cannot extend nearer the base circle than point A. The shaded portion of the wheel tooth is, therefore, useless when meshing with the 16-tooth pinion and could be removed by topping without causing any inconvenience. Less undercutting and interference would also result if the pressure angle were increased.



	ADD.	DED.
PINION	·155	·126
WHEEL	·095	·186

Fig. 15.

Improved tooth action and partial, if not the whole, elimination of interference and undercutting may be obtained by "correcting" the teeth. The teeth in this case are "corrected" by increasing the addendum and decreasing the dedendum of the pinion and correspondingly decreasing the addendum and increasing the dedendum of the wheel, the full depth of tooth in both instances remaining as standard. Fig. 15 shows the teeth "corrected" ·030 inch, and it will be seen that contact between the tip of the wheel and the flank of the pinion now takes place. One result of this is that the length of the line of contact has been increased, giving better tooth action and also, with the elimination of the undercutting, stronger teeth result.

If the working face of one tooth is divided into a number of equal parts, the part on the mating tooth contacting with any one portion may be indicated, bearing in mind that contact can only take place on the line of contact.

Referring to Fig. 15, the numbers on one tooth face correspond to the portions with which they work on the other tooth face. The divisions marked 5 on the pitch diameter are equal, therefore rolling takes place at this point. At the root of the pinion tooth, the working portion 1 is much less than the corresponding division on the wheel. This is indicative of sliding at the lower portions of the tooth flank causing the greatest wear at this point. This, unfortunately, is unavoidable, but may be reduced to a minimum by careful study of the tooth action and the teeth corrected if necessary to give improved action.

When the teeth are corrected, the centres remaining the same, the outside diameter of the pinion is increased by twice the increase in addendum correction, and the wheel blank decreased by a like amount. The teeth are then cut to standard depth. The pitch diameter remains unaltered when the teeth are corrected, this being as for a standard gear—

$$\frac{\text{Number of teeth}}{\text{Diametral pitch}}$$

The base circle diameter also remains unaltered. The rack form cutter pitch line still rolls on the pitch diameter without slip, but the position of the pitch line of the rack form cutter is modified to suit the correction, *i.e.*, the addendum of the rack type cutter is equal to the dedendum of the gear to be cut. When a gear tooth is positively corrected, *i.e.*, the addendum is increased and the dedendum decreased, the width of tooth on the rack type cutter, and therefore the tooth space on the gear, is decreased as shown in Fig. 16.

As the pitch line of the rack rolls on the pitch circle of the gear without slip, it follows that the circular width of tooth space on the

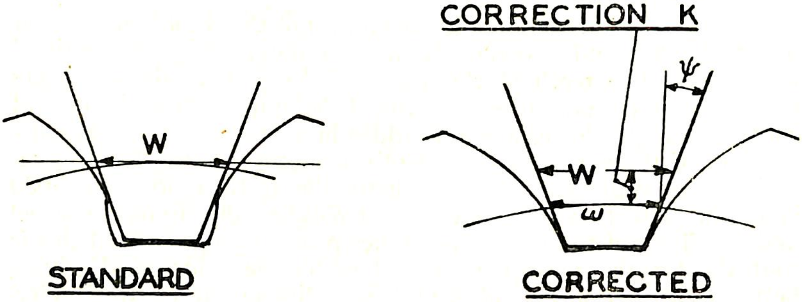


Fig. 16.

pitch circle of the gear is equal to the width of the rack tooth on the pitch line. If then, the gear is positively corrected, the cutter is withdrawn from the centre, the pitch line remaining.

When K_p = Positive correction.
K_n = Negative correction.
ψ = Pressure angle.
W = Width of rack tooth on pitch line for standard gear, *i.e.*, half circular pitch.
w = Width of rack tooth on pitch line—corrected gear.
then $w = W - 2\,(K_p \tan \psi)$
and $w = W + 2\,(K_n \tan \psi)$

The gear tooth circular width of the corrected gear on the pitch circle is therefore,

$$\text{Circular pitch} \pm w$$

By increasing the addendum of a standard gear, involute profile further away from the base circle is used. Too much correction will cause tooth pointing, a condition which must be avoided either by decreasing the correction or if such correction is necessary for tooth strength, etc., by topping the teeth.

It will be realised that corrected teeth may be cut with standard cutters when the teeth are generated either by rack shaping, hobbing, or Fellows cut, but special cutters are required for milled gear teeth.

Tooth correction, as illustrated in the foregoing, is such that the increase in the dedendum of the wheel to correspond to the increase in the addendum of the pinion, did not cause serious undercutting, or interference, in the wheel teeth. This was due to the, comparatively speaking, large number of teeth in the wheel.

When, however, two mating gears each have a small number of teeth, both must be positively corrected to give the best tooth form. Assume that two gears, each having a small number of teeth, are required to run together. The pitch diameters and base circle diameters are computed in the normal way from the number of teeth, diametral pitch and pressure angle of the cutter to be used. It is found that the gear of N teeth requires a positive correction of x and the gear of n teeth requires a positive correction of y to avoid undercutting and interference. As both gears need positive correction the theoretical centre distance of the gears must be increased to accommodate the increase in addenda of both gears. This increase is something less than the total amount of correction of both gears.

If the centres are increased by the total amount of correction of both gears, the backlash between the teeth will be excessive. Referring to Fig. 17, the pressure angle of the generating rack cutter is ψ. Contact with the teeth is therefore at A and B as the rack tooth is symmetrical with the rack space, and so diagrammatically the rack side shown may be taken as the cutting edge. But

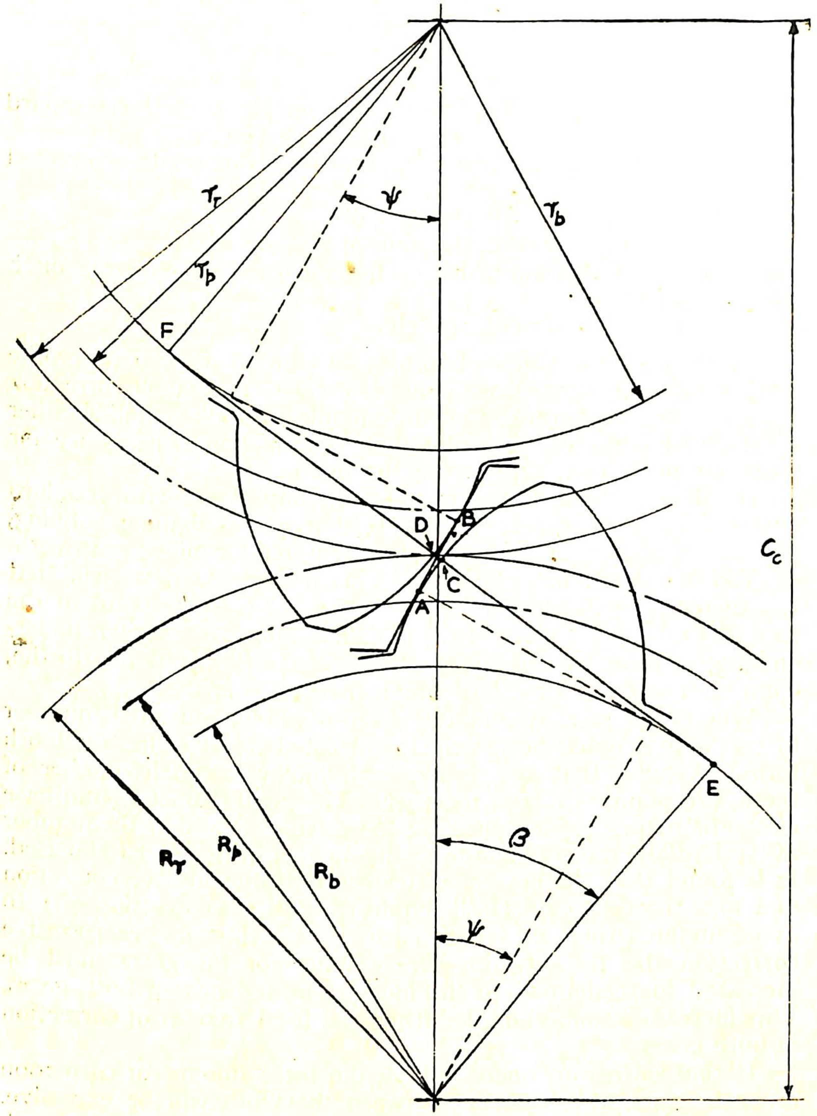


Fig. 17.

when running, contact between the teeth occurs at the new pitch point on the new line of action EF. With the centres extended by the amount of the total addendum correction for the two mating gears, the backlash on the new centres would be DC, which for gears of a small number of teeth would be excessive. The centres must therefore be closed in to give the required running backlash.

Assuming the centres have been extended by the total addenda correction, as shown in Fig. 17,

$$C_c = \frac{N + n}{2\,DP} + (x + y)$$

The contacting pitch radii are now R_r and r_r and as the velocity ratio has not been altered,

$$N/n = R_p/r_p = R_b/r_b = R_r/r_r$$

$$\text{and } R_r = \frac{C_c \times N}{N + n} \text{ and } r_r = \frac{C_c \times n}{N + n}$$

$$\text{Cos } \beta = R_b/R_r \text{ or } r_b/r_r$$

where N = Number of teeth in wheel.
n = Number of teeth in pinion.
x = Addendum correction of wheel.
y = Addendum correction of pinion.
DP = Diametral pitch.
C_c = Centre distance with centres extended by the total addenda correction.

The centres must now be closed in to eliminate the excessive backlash DC. (See "Gauging of Straight Spur Teeth" to ascertain centre distance with no backlash). In all the above calculations, running backlash has been ignored; for computation purposes it is assumed that the gears are running with no backlash. Having ascertained the centres for the corrected gears without backlash, the required running backlash may be cut into the gear in the normal way, *i.e.*, by feeding in the cutter. When the centres have been closed in to eliminate the excessive backlash, it may be found necessary to "top" the teeth owing to the reduced bottom clearance caused by the closing in of the centres.

Gauging of Straight Spur Gear Teeth.

The usual method for checking gear tooth thickness is by means of the gear tooth calliper, whereby the tooth thickness is measured at a definite distance from the outside diameter. The tooth thickness may be measured either—

(1) As the chord across from the points of intersection of the pitch circle and the tooth flanks. (The tooth thickness at this point is known as the "chordal thickness" and the height of the chordal thickness from the outside

diameter is known as the "corrected addendum"). For any given diametral pitch and pressure angle the "chordal thickness" and "corrected addendum" vary according to the number of teeth in the gear.

(2) By the constant chord setting. By this method only one calliper setting is required for a given diametral pitch and pressure angle, irrespective of the number of teeth in the gear, providing all are cut by the basic rack cutter to standard tooth proportions, or, all corrected by the same amount.

Referring to Fig. 18,

To find the chordal thickness.

For gears cut to standard proportions the circular thickness at the pitch radius (ignoring backlash)

$$= \frac{\text{Pitch diameter} \times \pi}{2 \times \text{No. of teeth}}$$

$$= \frac{\text{Circular pitch}}{2}$$

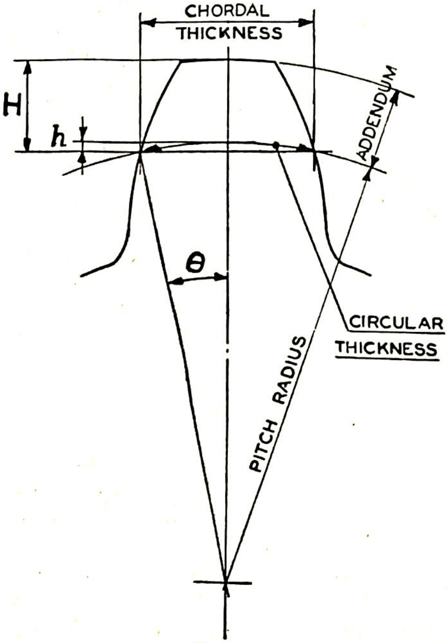


Fig. 18.

For corrected teeth, compute circular thickness at pitch line (see Correction of Spur Gear Teeth).

$$\theta \text{ (degrees)} = \frac{360 \times \text{Circular thickness}}{\text{Pitch diameter} \times \pi \times 2}$$

$$\textit{Chordal thickness} = 2 \times \text{Pitch radius} \times \sin \theta.$$

To find the corrected addendum H

$$h = \text{Pitch radius } (1 - \cos \theta).$$

Corrected addendum $= h +$ addendum.

Constant chord calliper setting.

Referring to Fig. 19, the circular thickness of the tooth on the pitch circle is equal to half the circular pitch (assuming no running backlash) and is also equal to the width of the tooth space on the pitch line of the rack. Under the condition shown, contact between the rack and the gear will always be at a and b, these points being normal to the rack sides and tangent to the base circle diameter; the two lines of action pass through the pitch point.

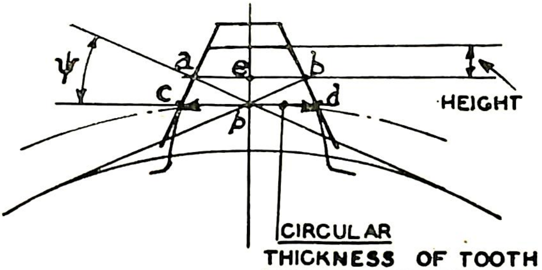


Fig. 19.

For the rack to contact at the pitch point it must move a distance of dp, i.e., a quarter of the circular pitch, and in moving this distance the point of contact has moved from b to p. For standard gears cut to the same circular thickness with the same basic rack of pressure angle ψ, a and b are fixed points relative to the pitch point, as also are the distances ap and bp. The chord ab, therefore, is also constant and is known as the "constant Chord." It also follows that there is a "constant chord" for all gears corrected by the same amount and cut by the same basic rack cutter.

$$bp = dp \cos \psi$$

Therefore $$ab = 2\, dp \cos \psi \cos \psi = 2\, dp \cos^2 \psi$$

$$\textit{Constant chord thickness} = \frac{\text{Circular pitch}}{2} \cos^2 \psi$$

$$bd = dp \sin \psi$$

$$ep = bd \cos \psi = dp \sin \psi \cos \psi$$

Constant chord height

$$= \text{Addendum} - \frac{\text{Circular pitch} \times \sin \psi \cos \psi}{4}$$

where $dp = \dfrac{\text{Circular pitch}}{4}$

ψ = Pressure angle.

Another convenient method of gauging the tooth thickness is to measure the distance over a circular pin inserted in the tooth space. Either a pin diameter may be selected such that contact between the pin and the tooth flank occurs on the pitch diameter or a predetermined diameter pin may be used, contact not necessarily occurring on the pitch line.

To find diameter of gauging pin to contact at pitch diameter and distance over pins.

Referring to Fig. 20,

$t/2\,R$ = Angle $O\hat{Q}A$. Convert to degrees.

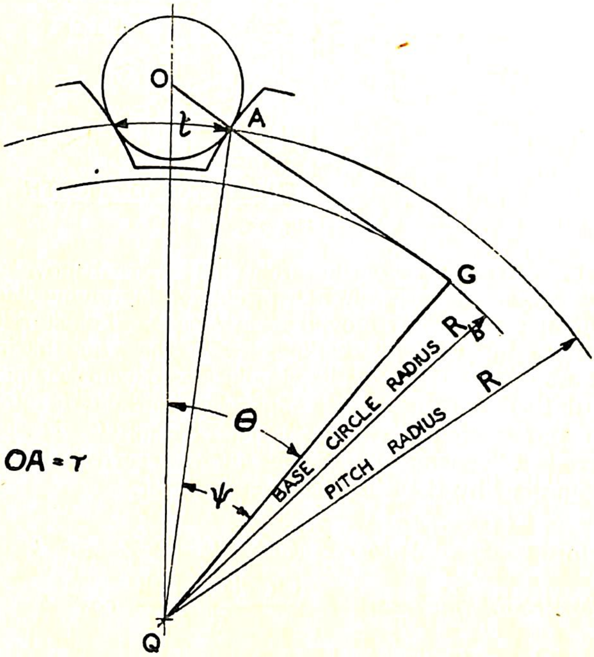


Fig. 20.

$$\text{Angle } O\hat{Q}A + \psi = \theta$$

$$OG = R_b \tan \theta$$

$$AG = R_b \tan \psi$$

Therefore $r = OG - AG = R_b (\tan \theta - \tan \psi)$

Dia. of pin $= 2\, R_b (\tan \theta - \tan \psi)$

$$OQ = R_b / \cos \theta$$

Therefore—*Distance over pins.* (Even number of teeth)

$$= 2\,(OQ + r)$$

Referring to Fig. 21. (Odd number of teeth)

$$= 2\left[\left(OQ \times \cos \frac{90}{N}\right) + r\right]$$

where N = number of teeth.

(Angle subtended by ·25 circular pitch = 90/N).

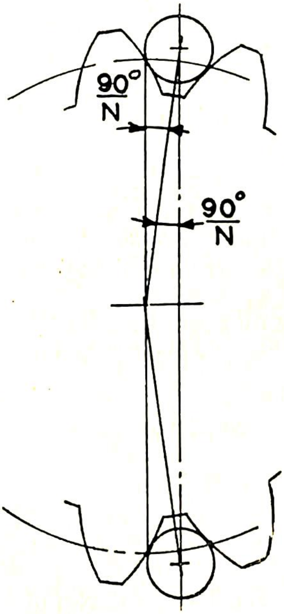


Fig. 21.

When gauging over pins of a pre-determined diameter, the same formula is used, irrespective of the number of teeth in the gear. The derivation of the formula, however, varies slightly, depending on the number of teeth, which in turn decides which side of the centre of the tooth space is the origin of the involute. The two derivations of the formula are—

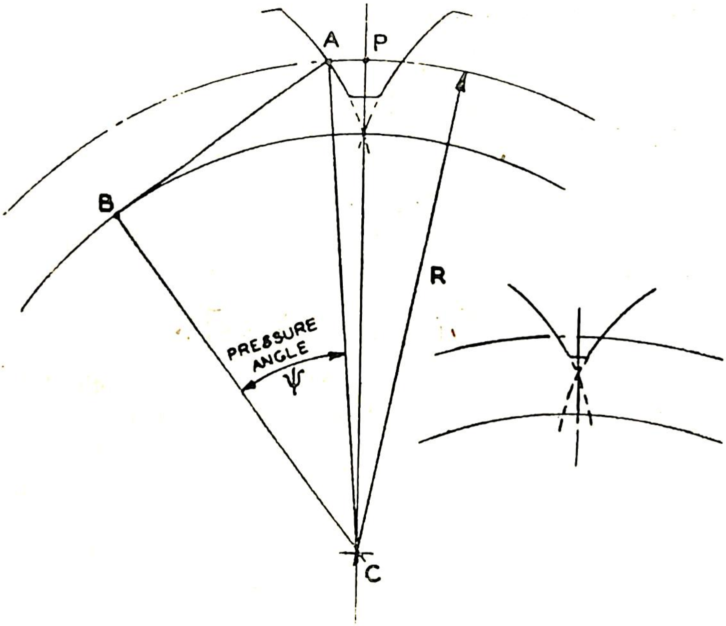


Fig. 22.

(1) Gears having a number of teeth such that the origin of the involute does not cross the centre line of the tooth space, and

(2) Gears having a number of teeth such that the origin of the involute does cross the centre line of the tooth space.

Referring to Fig. 22,

$$AP = \frac{\text{Circular width of space}}{2}$$

$$= \frac{2\ \pi\ R}{2\,N \times 2} = \pi\ R/2\,N$$

$$\text{Angle } A\hat{C}P = AP/R = \pi/2\,N \text{ radians.}$$

$$\text{Angle } A\hat{C}P = \text{inv. } \psi$$

$$\pi/2\,N = \text{inv. } \psi \quad \text{Therefore } N = \frac{\pi}{2 \text{ inv. } \psi}$$

Thus if number of teeth is $\frac{\pi}{2 \text{ inv. } \psi}$ the origin of the involute is on the point of intersection of the centre line of the tooth space.

If the number of teeth exceeds $\dfrac{\pi}{2 \text{ inv. } \psi}$ the involute curve crosses the centre line of the tooth space from its origin. (See inset).

To find distance over known diameter pins (for gears having a number of teeth of $\dfrac{\pi}{2 \text{ inv. } \psi}$ or under).

Referring to Fig. 23,

$AO = r =$ Radius of pin.

$r =$ Arc ED Angle $E\hat{Q}D = r/R_b$

Angle $CQP = t/2R$ Angle $CQE =$ inv. ψ

$$\text{Angle } E\hat{Q}F = \text{Angle } C\hat{Q}P - \text{Angle } C\hat{Q}E$$
$$= t/2R - \text{inv. } \psi$$

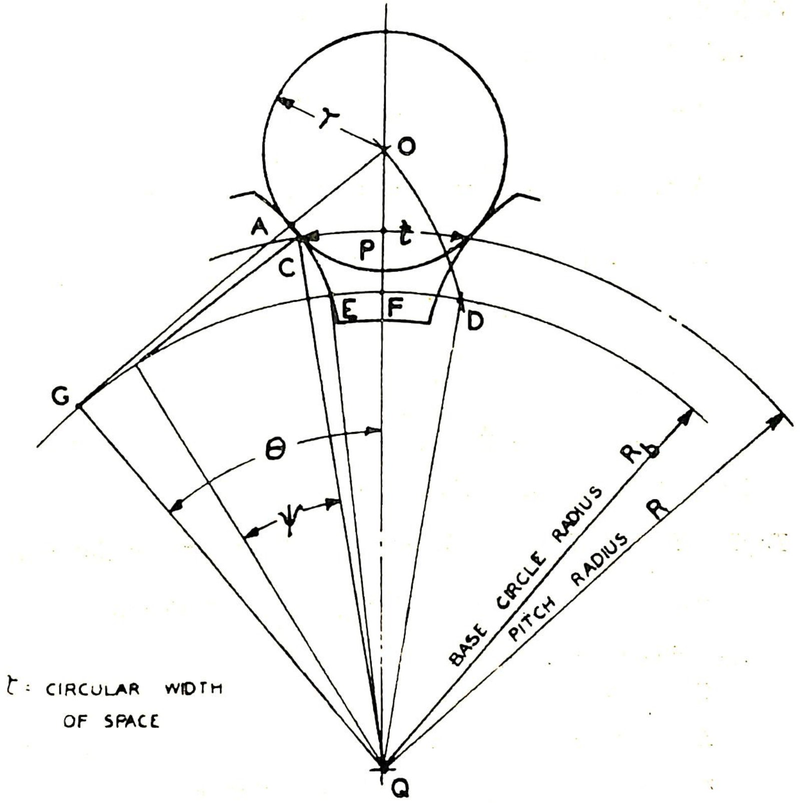


Fig. 23.

$$\text{Angle } F\hat{Q}D = \text{Angle } E\hat{Q}D - \text{Angle } E\hat{Q}F = \text{inv. } \theta$$

$$\text{Inv. } \theta = \frac{r}{R_b} - \frac{t}{2\,R} + \text{inv. } \psi$$

Find θ in degrees. $OQ = R_b/\cos\theta$

Therefore *distance over pins*—

(Even number of teeth)

$$= 2\,(OQ + r)$$

(Odd number of teeth)

$$= 2\left[\left(OQ \times \cos\frac{90}{N}\right) + r\right]$$

For gears having a number of teeth of over $\dfrac{\pi}{2\text{ inv. }\psi}$ the derivation of the formula is slightly modified.

Referring to Fig. 24,

$r = \text{Arc } ED$, $\qquad \text{Angle } E\hat{Q}D = \dfrac{r}{R_b}$

$\text{Angle } C\hat{Q}P = \dfrac{t}{2\,R} \qquad \text{Angle } C\hat{Q}E = \text{inv. } \psi$

$$\text{Angle } E\hat{Q}F = \text{Angle } C\hat{Q}E - \text{Angle } C\hat{Q}P$$

$$= \text{inv. } \psi - \frac{t}{2\,R}$$

$$\text{Angle } F\hat{Q}D = \text{Angle } E\hat{Q}F + \text{Angle } E\hat{Q}D = \text{inv. } \theta$$

$$\text{Inv. } \theta = \text{inv. } \psi - \frac{t}{2\,R} + \frac{r}{R_b}$$

Proceed then as for previous case.

To find circular width of tooth, knowing distance over pins (for gears having a number of teeth of $\dfrac{\pi}{2\text{ inv. }\psi}$ or under).

Referring to Fig. 23,

$GQ/OQ = \text{Cos } \theta \qquad$ Find θ in degrees.

$\text{Angle } F\hat{Q}D = \text{inv. } \theta$

$r = \text{Arc } ED \qquad \text{Angle } E\hat{Q}D = r/R_b$

$\text{Angle } E\hat{Q}F = E\hat{Q}D - F\hat{Q}D$

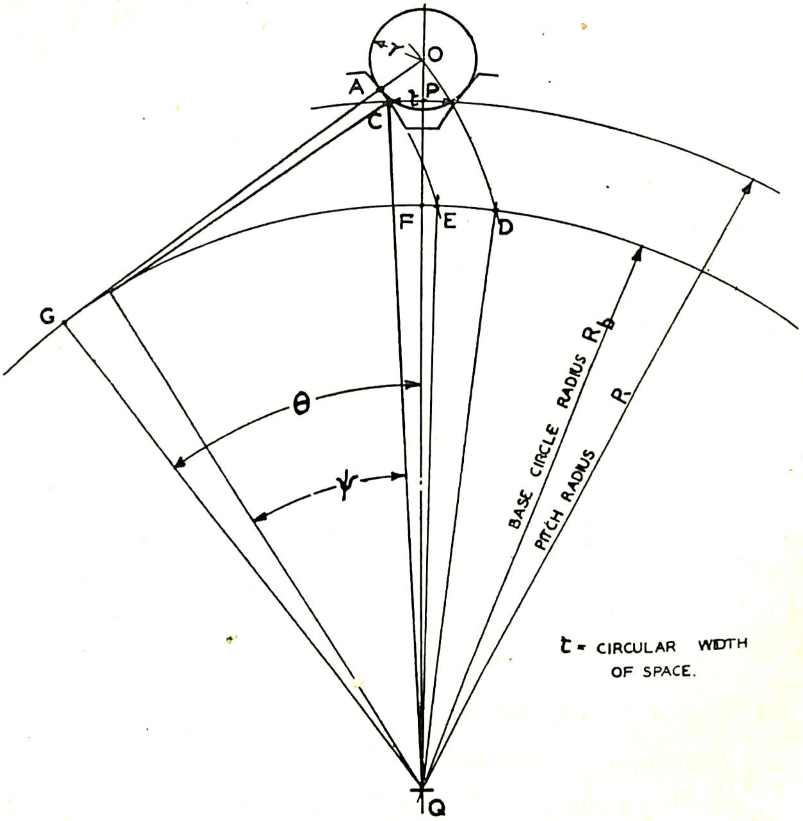


Fig. 24.

$$= \frac{r}{R_b} - \text{inv. } \theta$$

$$\text{Angle } C\hat{Q}E = \text{inv. } \psi$$

$$\text{Angle } C\hat{Q}P = C\hat{Q}E + E\hat{Q}F$$

$$= \frac{r}{R_b} + \text{inv. } \psi - \text{inv. } \theta$$

Therefore t (circular width of tooth space)

$$= 2\,R\left(\frac{r}{R_b} + \text{inv. } \psi - \text{inv. } \theta\right)$$

Circular width of tooth

= Circular pitch − t

$$= \frac{2\pi R}{N} - 2R\left(\frac{r}{R_b} + \text{inv. } \psi - \text{inv. } \theta\right)$$

$$= 2R\left(\frac{\pi}{N} - \frac{r}{R_b} - \text{inv. } \psi + \text{inv. } \theta\right)$$

Again, for gears having a number of teeth of over $\frac{\pi}{2 \text{ inv. } \psi}$ the derivation of the formula is slightly modified.

Referring to Fig 24,

$GQ/OQ = \text{Cos } \theta$ Find θ in degrees.

Angle $F\hat{Q}D = \text{inv. } \theta$

$r = \text{Arc } ED$ Angle $E\hat{Q}D = r/R_b$

Angle $F\hat{Q}E = F\hat{Q}D - E\hat{Q}D$

$$= \text{inv. } \theta - \frac{r}{R_b}$$

Angle $C\hat{Q}E = \text{inv. } \psi$

Angle $C\hat{Q}P = C\hat{Q}E - F\hat{Q}E$

$$= \text{inv. } \psi - \text{inv. } \theta + \frac{r}{R_b}$$

Proceed then as for previous case.

To find distance across two or more teeth.

Tooth spacing combined with tooth thickness and possible profile errors may be gauged by the known distance across two or more teeth.

Referring to Fig. 25,

Angle $A\hat{O}B = t/R$

Arc BC = Circular pitch at pitch radius R × number of tooth spaces.

$$= \frac{2\pi R n}{N}$$

Angle $B\hat{O}C = \frac{2\pi R n}{N R} = \frac{2\pi n}{N}$

Angle $D\hat{O}A$ and $C\hat{O}E = \text{inv. } \psi$

Angle $D\hat{O}E = D\hat{O}A + A\hat{O}B + B\hat{O}C + C\hat{O}E$

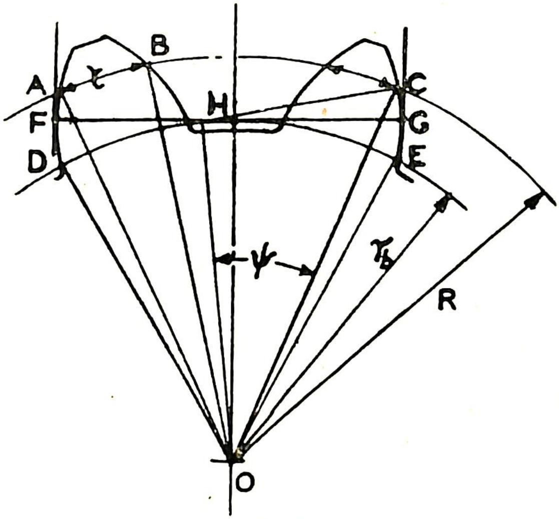


Fig. 25.

$$= \text{inv. } \psi + \frac{t}{R} + \frac{2\pi n}{N} + \text{inv. } \psi$$

$$= 2\,\text{inv. } \psi + \frac{t}{R} + \frac{2\pi n}{N}$$

Arc DHE = Angle $D\hat{O}E \times r_b$
Arc DH = Line FH
Arc HE = Line HG

FHG is a straight line tangent to the base circle.

Therefore F and G are the points of contact, and

FG = Arc DHE

$$FG = r_b \left(2\,\text{inv. } \psi + \frac{t}{R} + \frac{2\pi n}{N}\right)$$

Thus distance across two or more teeth

$$= r_b \left(2\,\text{inv. } \psi + \frac{t}{R} + \frac{2\pi n}{N}\right)$$

where t = Circular thickness of tooth at pitch radius.
r_b = Base circle radius.
R = Pitch radius.
n = Number of tooth spaces between teeth to be gauged.
N = Number of teeth in gear.

Note the gauging line FHG need not necessarily be at right angles to HO. FHG is a straight line and normal to the involute curves at points F and G, therefore the gauging line is always tangential to the base circle.

To find the centre distance at which a pair of mating gears will mesh without backlash.

Another useful gauging method is to check the gears running on closed-in centres with no backlash. The following computation will also be found useful for ascertaining the centre correction to eliminate the excessive backlash when running two positively corrected gears on extended centres.

Referring to Fig. 26,

$$t_p = 2\, r_p \left(\frac{t}{2\, r} + \text{inv. } \psi - \text{inv. } \beta \right)$$

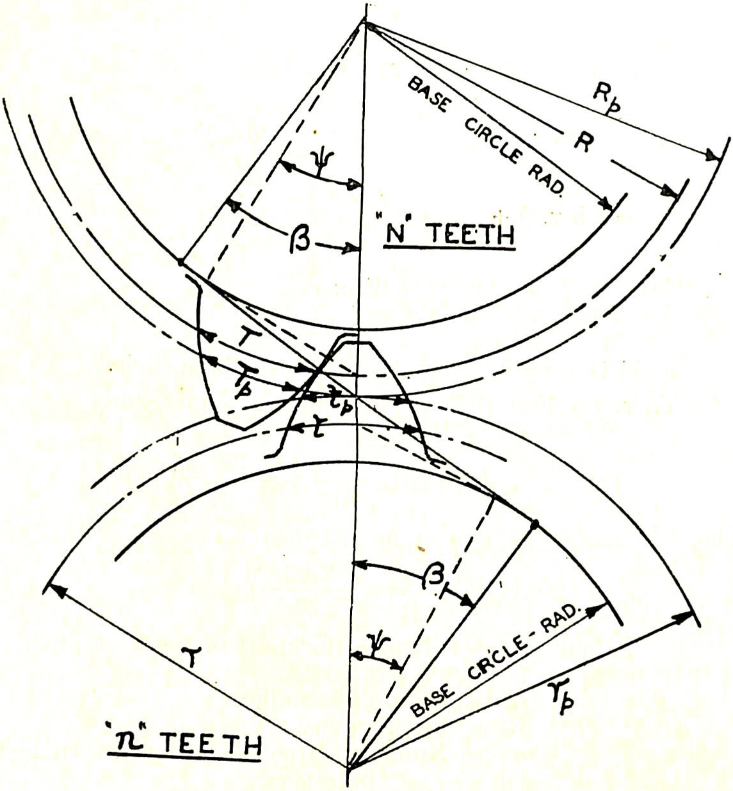


Fig. 26.

$$= \frac{r_p\, t}{r} + 2\, r_p\, (\text{inv. } \psi - \text{inv. } \beta)$$

$$T_p = \frac{2\, r_p\, N}{n}\left(\frac{Tn}{2\, r\, N} + \text{inv. } \psi - \text{inv. } \beta\right)$$

$$= \frac{T\, r_p}{r} + \frac{2\, r_p\, N}{n}\, (\text{inv. } \psi - \text{inv. } \beta)$$

Therefore $t_p + T_p$

$$= \frac{r_p\,(T + t)}{r} + \frac{2\, r_p\,(N + n)\,(\text{inv. } \psi - \text{inv. } \beta)}{n}$$

But $t_p + T_p$ = Circular pitch

$$= \frac{2\, \pi\, r_p}{n}$$

Therefore

$$\frac{2\, \pi\, r_p}{n} = \frac{r_p\,(T + t)}{r} + \frac{2\, r_p\,(N + n)\,(\text{inv. } \psi - \text{inv. } \beta)}{n}.$$

Divide by r_p :

$$\frac{2\, \pi}{n} = \frac{(T + t)\, n}{r\, n} + \frac{2\,(N + n)\,(\text{inv. } \psi - \text{inv. } \beta)}{n}$$

Multiply by n :

$$2\, \pi = \frac{(T + t)\, n}{r} + 2\,(N + n)\,(\text{inv. } \psi - \text{inv. } \beta)$$

$$\text{or } 2\, \pi - \frac{(T + t)\, n}{r} = 2\,(N + n)\,(\text{inv. } \psi - \text{inv. } \beta)$$

$$\text{or} \quad \frac{2\, \pi\, r - (T + t)\, n}{2\, r\,(N + n)} = \text{inv. } \psi - \text{inv. } \beta$$

Therefore

$$\text{Inv. } \beta = \frac{n\,(T + t) - 2\, \pi\, r}{2\, r\,(N + n)} + \text{inv. } \psi$$

From the above, determine β in degrees.

Theoretical centre distance for pressure angle $\psi = R + r$

Therefore centre distance for pressure angle β

$$= R_p + r_p = \frac{(R + r)\cos \psi}{\text{Cos } \beta}$$

IV.—HELICAL GEARS.

If the action of mating straight spur gears going into and out of mesh is studied, it will be seen that it is far from ideal from the point of view of wear and consequent noise. The tip of the tooth enters into engagement along its entire length and is therefore subjected to a shock load, which, however, is slightly alleviated by a slight distortion of the tooth tip. At the point of entry, the entering tooth tip slides over the flank of the mating tooth adjacent to the fillet radius. The next stage of engagement is at the pitch point where rolling between the mating teeth occurs. The last stage of engagement is where the flank of the tooth adjacent to the fillet radius is contacting with the tip of the mating tooth. Sliding again takes place. It is apparent that the tooth loading of straight spur gears is such to cause a wide variation of stress in the tooth throughout the arc of action, and much of the loading is in the nature of shock loads. Such loads, together with the sliding at the flank adjacent to the fillet radius, induce wear where the involute profile is least able to withstand it. (When two curves touch, the load-carrying capacity is dependent on the radii of curvature, and with the involute profile, the instant radius decreases as the base circle diameter is approached).

The action of the helical gear is somewhat different. Due to the angle of the teeth to the axis (the helix angle) the whole length of the tooth tip does not enter into engagement at the same time. The sequence of engagement is the same as for a straight spur, but at no time are the tooth tips or flanks adjacent to the fillet radius loaded along their entire length. The contact line of a helical gear tooth runs diagonally across the tooth face and not parallel as with a spur. Helical tooth loading is, therefore, much more gradual and shock loading practically eliminated. Maintenance of involute profile and consequently quietness of running are inherent features of the helical gear.

The angle of the helical tooth to the axis, *i.e.*, the helix angle, causes a side thrust to be set up. With single helicals the side thrust must be accommodated, but with double helicals or herring-bone gears (which are virtually two identical single helices side by side but with opposite hand helices) the side thrusts set up in the gear cancel out. The end thrust may be computed from

$$\text{End Thrust} = \frac{\text{Torque} \times \text{Tan helix angle}}{\text{Pitch radius of gear}}$$

It will be seen that with single helical gears, the end thrust is proportional to the tangent of the helix angle, which should therefore be kept as small as other considerations will allow. With double helicals, where the end thrust is self-contained, this is not so important. In instances where more than one single helical gear is mounted on the same shaft and in operation at the same time, careful selection of the helix angles and helix hands will enable the respective end thrusts to wholly or partially cancel out each other.

To take the greatest advantage of the helical gear, the helix should be such that the trailing tip of one tooth should not leave engagement before the leading tip of the following tooth, *i.e.*, ratio of the helical advance to the circular pitch in the diametral plane should be greater than unity.

$$\text{Helical advance Ratio} = \frac{\text{Tan helix angle} \times \text{Face width}}{\text{Circular pitch in diametral plane}}$$

The helical advance increases with the tangent of the helix angle, as also does the end thrust with single helical gears. It is therefore necessary to compromise between a minimum end thrust and a helical advance of from 1·0 to 1·15. Where the contact ratio is high, as with gears of a very large number of teeth, the helical advance ratio may be reduced to below unity.

Assuming the gear to be of standard proportions in the normal plane, the tooth width and space will be greater in the diametral plane, so that in this plane the circular pitch is also greater. Hence for a given pitch and pitch diameter, the helical gear has less teeth than the corresponding straight spur, the number of teeth decreasing as the helix angle increases. The ratio between the number of teeth in the helical gear to the number of teeth in the corresponding straight spur is as the cos of the helix angle.

Hence,

$$\text{Cos helix angle} = \frac{N + n}{p \times 2C}$$

where N = Number of teeth in wheel.
n = Number of teeth in pinion.
p = Diametral pitch (normal plane).
C = Centre distance.

The helix angle is understood to be taken at the pitch radius, unless otherwise stated. As will be shown later, for a specified gear, the helix angle varies, dependent on the radius at which it is taken.

Dimensions of helical gears are referred to as being in one of two planes. The normal plane, *i.e.*, the plane normal to the tooth face, and the diametral plane, *i.e.*, the plane of rotation. If a standard pitch cutter is used, this will cut standard tooth proportions in the normal plane, irrespective of the helix angle. The tooth width in the diametral plane, however, increases as the helix angle increases. Alternatively, if a cutter is used giving standard tooth proportions in the diametral plane, the tooth width in the normal plane decreases in width as the helix angle is increased. With hobbed gears of the same normal pitch, the same cutter may be used irrespective of the helix angle. This favours the system of maintaining standard tooth proportions in the normal plane—the standard tooth proportions being as for a straight spur gear.

From the following formulae, the proportions of the required gear may be calculated (all dimensions in inches) :—

$$\text{(1) Circular pitch (diametral plane)} = \frac{\text{Circular pitch (normal plane)}}{\text{Cos helix angle}}$$

$$\text{(2) Circular pitch (diametral plane)} = \frac{\text{Pitch diameter} \times \pi}{\text{No. of teeth}}$$

$$\text{(3) Circular pitch (diametral plane)} = \frac{\pi}{\text{Diametral pitch (normal plane)} \times \text{Cos helix angle}}$$

$$\text{(4) Diametral pitch (diametral plane)} = \text{Diametral pitch (normal plane)} \times \text{Cos helix angle}$$

$$\text{(5) Diametral pitch (diametral plane)} = \frac{\text{Number of Teeth}}{\text{Pitch diameter}}$$

$$\text{(6) Diametral pitch (diametral plane)} = \frac{\pi}{\text{Circular pitch (diametral plane)}}$$

$$\text{(7) Circular pitch (normal plane)} = \frac{\pi}{\text{Diametral pitch (normal plane)}}$$

$$\text{(8) Circular pitch (normal plane)} = \frac{\text{Pitch dia.} \times \pi \times \text{Cos helix angle}}{\text{Number of teeth}}$$

$$\text{(9) Diametral pitch (normal plane)} = \frac{\text{Number of teeth}}{\text{Pitch dia.} \times \text{Cos helix angle}}$$

The addendum, clearance and dedendum are the same in both the normal and diametral planes, and are as for a straight spur gear of the same pitch.

$$\text{(10) Circular thickness of tooth (normal plane)} = \frac{\text{Circular pitch (normal plane)}}{2}$$

(From the above must be deducted the running backlash).

$$\text{(11) Circular width of tooth space (normal plane)} = \frac{\text{Circular pitch (normal plane)}}{2}$$

(To the above must be added the running backlash).

$$\text{(12) Circular thickness of tooth (diametral plane)} = \frac{\text{Circ. thickness of tooth (normal plane)}}{\text{Cos helix angle}}$$

(13) Circular width of tooth space (diametral plane)

$$= \frac{\text{Cir. width of tooth space (normal plane)}}{\text{Cos helix angle}}$$

The pitch diameter and base circle diameter are taken in the diametral plane, as in this plane—the plane of rotation—both these are true diameters, whereas in the normal plane both are ellipses.

(14) Pitch dia. $= \dfrac{\text{No. of teeth} \times \text{Circular pitch (diam. plane)}}{\pi}$

(15) Pitch dia. $= \dfrac{\text{Number of teeth}}{\text{Diametral pitch (normal plane)} \times \text{Cos helix angle}}$

(16) Base Circle diameter = Pitch diameter × Cos pressure angle (diametral plane)

(17) Base pitch (diametral plane) = Circular pitch (diametral plane) × Cos pressure angle (diametral plane)

(18) Referring to Fig. 27,

In normal plane, $b = a \times \tan \psi_n$

In pitch line plane, $c = a \times \cos \alpha$

$$\text{Therefore} \quad \frac{b}{c} = \frac{a \times \tan \psi_n}{a \times \cos \alpha} = \tan \psi_d$$

$$\text{Tan pressure angle (diametral plane)} = \frac{\text{Tan pressure angle (nor. plane)}}{\text{Cos helix angle}}$$

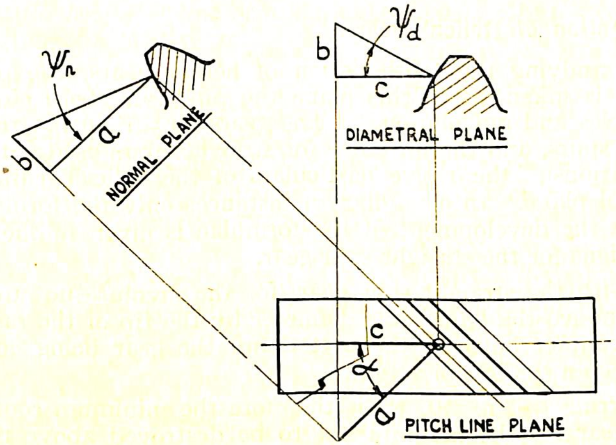


Fig. 27.

The pressure angle in the normal plane at the pitch point being the pressure angle of the cutter.

(19) Number of teeth = Pitch dia. × diametral pitch (normal plane) × cos helix angle.

(20) Centre distance (external gears) $= \dfrac{N + n}{2\,p_n} \times \dfrac{1}{\text{Cos helix angle}}$

(21) Centre distance (internal gears) $= \dfrac{N - n}{2\,p_n} \times \dfrac{1}{\text{Cos helix angle}}$

where N = Number of teeth in wheel.
n = Number of teeth in pinion.
p_n = Diametral pitch (normal plane).

(22) The lead of the helix is the amount of advance along the axis for one complete revolution.

Lead of helix = Pitch dia. × π × Cot. helix angle.

(23) Lead of helix $= \dfrac{\text{No. of teeth} \times \pi \times \text{Cosec helix angle}}{\text{Diametral pitch (normal plane)}}$

The lead is constant for any specified gear, irrespective of the diameter at which it is taken. Knowing the helix angle at a certain radius, the helix angle at any other radius may be found.

$$\text{Cot } \beta = \frac{R \times \cot \alpha}{r}$$

where α = Helix angle at radius R.
β = Helix angle at radius r.

Tooth Action of Helical Gears.

For studying the tooth action of helical gears, the plane of rotation is taken, as in this plane the pitch and base circles are true circles and not ellipses. The gears may then be treated as straight spurs, and the formulae for straight spurs used with slight modifications; these give particulars of the helical gears in the diametral plane. In a number of instances only the formulae are given as the development of the formulae is given in the similar applications for the straight spur gear.

As with the straight spur gear, for the involute not to be destroyed above the base circle diameter by the tip of the rack form cutter, contact between the cutter and the gear being cut must always be on the line of action.

Referring to Fig. 10, R_r is therefore the minimum root radius of the gear for the involute not to be destroyed above the base circle diameter.

$R_r = R_b \cos \psi$

But $R_b = R_p \cos \psi$ Therefore $R_r = R_p \cos^2 \psi$

Dedendum $= R_p (1 - \cos^2 \psi)$

Dedendum (Std.) $= 1{\cdot}157/p_n$

Dedendum (Fellows) $= 1{\cdot}25/p_n$

$R_p = N/2\,p_{dn}$ But $p_{dn} = p_n \times \cos \alpha$

$$\text{Therefore } R_p = \frac{N}{2\,p_n \cos \alpha}$$

$$\frac{N}{2\,p_n \cos \alpha}(1 - \cos^2 \psi) = 1{\cdot}157/p_n \text{ (Std.) or } 1{\cdot}25/p_n \text{ (Fellows).}$$

$$\text{Therefore } N = \frac{2{\cdot}314 \cos \alpha}{1 - \cos^2 \psi} \text{ (Std.)}$$

$$\text{or } N = \frac{2{\cdot}5 \cos \alpha}{1 - \cos^2 \psi} \text{ (Fellows)}$$

N = Minimum number of teeth for involute not to be destroyed above the base circle.

where R_p = Pitch radius.
R_b = Base radius.
R_r = Root radius.
ψ = Pressure angle in **diametral** plane.
p_n = Diametral pitch in normal plane.
p_{dn} = Diametral pitch in diametral plane.
α = Helix angle.

To find the length of line of contact and contact ratio between two mating helical gears.

Referring to Fig. 11,

Length of line of contact

$$= \sqrt{r_0{}^2 - r_b{}^2} + \sqrt{R_0{}^2 - R_b{}^2} - (r_b + R_b) \tan \psi$$

$$\text{Contact ratio} = \frac{\text{Length of line of contact}}{\text{Base pitch}}$$

$$= \frac{\sqrt{r_0{}^2 - r_b{}^2} + \sqrt{R_0{}^2 - R_b{}^2} - (r_b + R_b) \tan \psi}{\text{Circular pitch (diametral plane)} \times \cos \psi}$$

where R_0 = Outside radius of wheel.
R_b = Base radius of wheel.
r_0 = Outside radius of pinion.
r_b = Base radius of pinion.
ψ = Pressure angle in **diametral** plane.

To find the length of line of contact and contact ratio between a helical gear and mating rack.

Referring to Fig. 12,

Length of line of contact

$$= A \operatorname{cosec} \psi + \sqrt{R_0^2 - R_b^2} - R_b \tan \psi$$

$$\text{Contact ratio} = \frac{\text{Length of line of contact}}{\text{Base pitch}}$$

$$= \frac{A \operatorname{cosec} \psi + \sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{\text{Circular pitch (diametral plane)} \times \cos \psi}$$

where R_0 = Outside radius of gear.
R_b = Base radius of gear.
A = Addendum of rack.
ψ = Pressure angle in **diametral** plane.

To find the length of line of contact and contact ratio between a helical gear and internal gear.

Referring to Fig. 13,

Length of line of contact

$$= (R_b - r_b) \tan \psi - \sqrt{R_1^2 - R_b^2} + \sqrt{r_0^2 - r_b^2}$$

$$\text{Contact ratio} = \frac{\text{Length of line of contact}}{\text{Base pitch}}$$

$$= \frac{(R_b - r_b) \tan \psi - \sqrt{R_1^2 - R_b^2} + \sqrt{r_c^2 - r_b^2}}{\text{Circular pitch (diametral plane)} \times \cos \psi}$$

where R_1 = Internal radius of internal gear.
R_b = Base radius of internal gear.
r_0 = Outside radius of gear.
r_b = Base radius of gear.
ψ = Pressure angle in **diametral** plane.

To find the arcs of approach and recession of two mating helical gears.

Referring to Fig. 11,

Arc of approach of gear A

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{\operatorname{Cos} \psi}$$

Arc of approach of gear A (radians)

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{r_b}$$

Arc of recession of gear A

$$= \frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{\text{Cos } \psi}$$

Arc of recession of gear A (radians)

$$= \frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{r_b}$$

where R_0 = Outside radius of wheel.
R_b = Base radius of wheel.
r_0 = Outside radius of pinion.
r_b = Base radius of pinion.
ψ = Pressure angle in **diametral** plane.

To find arcs of approach and recession of a helical gear and mating rack.

Referring to Fig. 12, assuming gear to be the driver,

Arc of approach of gear

$$= \frac{A \text{ cosec } \psi}{\text{Cos } \psi}$$

Arc of approach of gear (radians)

$$= \frac{A \text{ Cosec } \psi}{R_b}$$

Arc of recession of gear

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{\text{Cos } \psi}$$

Arc of recession of gear (radians)

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{R_b}$$

where R_0 = Outside radius of gear.
R_b = Base radius of gear.
A = Addendum of rack.
ψ = Pressure angle in **diametral** plane.

To find arcs of approach and recession of a helical gear and internal gear.

Referring to Fig. 13, assuming the gear to be the driver,

Arc of approach of gear

$$= \frac{R_b \tan \psi - \sqrt{R_1^2 - R_b^2}}{\text{Cos } \psi}$$

Arc of approach of gear (radians)

$$= \frac{R_b \tan \psi - \sqrt{R_1^2 - R_b^2}}{r_b}$$

Arc of recession of gear

$$\frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{\text{Cos } \psi}$$

Arc of recession of gear (radians)

$$\frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{r_b}$$

where R_1 = Internal radius of internal gear.
R_b = Base radius of internal gear.
r_0 = Outside radius of gear.
r_b = Base radius of gear.
ψ = Pressure angle in **diametral** plane.

Correction of Helical Gears.

Helical gears of a small number of teeth are subjected to undercutting and interference in the same manner as straight spur gears. For the same number of teeth, however, the helical gear is less prone to this trouble than the straight spur.

Referring to Fig. 28, in the normal plane of a helical gear the pitch diameter is an ellipse, and the instant radius of the ellipse at point A is very closely approximate to $r/\text{Cos}^2\,\alpha$. The tooth form is therefore as for a spur gear of D_v pitch diameter. This diameter is known as the "virtual diameter."

$$D_v = D/\cos^2 \alpha$$

Virtual number of teeth N_v (obtained from pitch dia.)

$$= D\, p_n/\text{Cos}^2\, \alpha$$

Diametral pitch in normal plane $p_n = \dfrac{N}{D \times \cos \alpha}$

Therefore, virtual number of teeth N_v (obtained from number of teeth)

$$= \frac{D \times N}{\text{Cos}^2\, \alpha \times D \times \cos \alpha}$$

$$= N/\cos^3 \alpha = N_v$$

where D = Pitch diameter.
D_v = Virtual pitch diameter.
α = Helix angle.
p_n = Diametral pitch in normal plane.
N = Number of teeth in gear.
N_v = Virtual number of teeth.

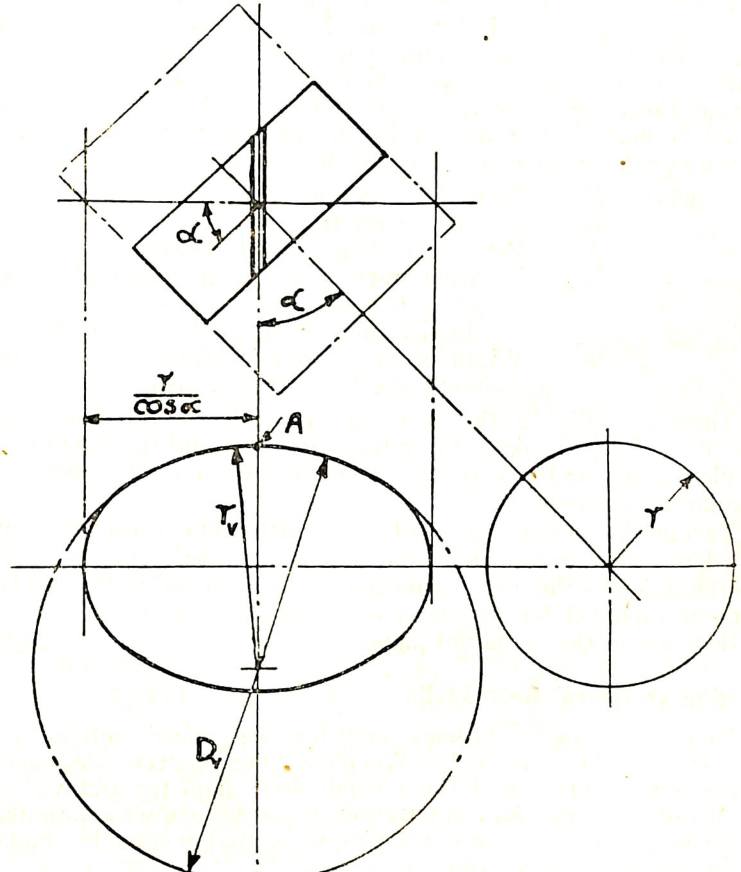


Fig. 28.

The tooth form in the normal plane of a helical gear of N teeth is therefore as for a spur gear of a virtual number of teeth N_v. It will be seen that with a helical gear a smaller number of teeth may be employed before there is undercutting and interference, and the larger the helix angle, the smaller the number of teeth possible.

The methods of correction for straight spur gears equally apply to helical gears. A helical gear may be said to be a large number of thin laminae, each a straight spur gear, each lamina staggered relative to the next, so forming the helix. Any tooth layouts and computations, and problems connected with the involute tooth should be made in the diametral plane and the tooth proportions, pressure angle, etc., converted to the diametral plane.

when K_p = Positive correction.
K_n = Negative correction.
ψ_d = Pressure angle in diametral plane.
W = Width of rack tooth in diametral plane on pitch line for standard gear, *i.e.*, half circular pitch in diametral plane.
w = Width of rack tooth in diametral plane on pitch line for corrected gear.

Then $w = W - 2\ (K_p \tan \psi_d)$ for positive correction and $w = W + 2\ (K_n \tan \psi_d)$ for negative correction and the gear tooth circular width of the corrected gear on the pitch diameter is—Circular pitch minus w.

Two mating helical gears, both of a small number of teeth, may each be positively corrected and run on extended centres. The determination of the new centres may be ascertained in the similar manner employed for straight spur gears; again, all tooth proportions are in the diametral plane.

Gauging of Helical Gear Teeth.

Helical gear tooth thickness may best be gauged and checked by the tooth calliper method. When checking, the teeth are gauged in the normal plane and the virtual pitch diameter and virtual number of teeth used for computation purposes. Only the formulae are given as the stages for developing same are given in the similar application for straight spur gears.

Referring to Fig. 18,

To find the Chordal Thickness.

For standard gears, the circular thickness at the pitch diameter, ignoring backlash

$$= \frac{D_v \times \pi}{2 \times N_v}$$

$$= \frac{\text{Circular pitch (normal plane)}}{2}$$

For corrected teeth, compute circular thickness at the pitch diameter—see "Correction of Helical Gear Teeth."

$$\text{Then } \theta \text{ (degrees)} = \frac{360 \times \text{Circular thickness at } D_v}{D_v \times \pi \times 2}$$

$$\text{Chordal thickness} = 2 \times R_v \times \sin \theta$$

where D_v = Virtual pitch diameter.
N_v = Virtual number of teeth.
R_v = Virtual pitch radius.

To find the Corrected Addendum.

$$h = R_v\ (1 - \cos \theta)$$

Corrected addendum, H = h + addendum.

Constant Chord Calliper Setting.

Referring to Fig. 19,

Constant chord thickness

$$= \frac{\text{Circular pitch (normal plane)} \cos^2 \psi_n}{2}$$

Constant chord height

$$= \text{Addendum} - \frac{\text{Circular pitch (normal plane)}}{4} \sin \psi_n \cos \psi_n$$

where ψ_n = Pressure angle in normal plane.

It also follows that there is a constant chord setting for all gears corrected by the same amount and cut by the same cutter.

Gear tooth thickness may also be gauged by running the mating gears on closed-in centres with no backlash.

Referring to Fig. 26,

$$\text{Inv. } \beta = \frac{n\ (T + t) - 2\ \pi r}{2\ r\ (N + n)} + \text{inv. } \psi$$

From the above, determine the meshing pressure angle β.
Theoretical centre distance for pressure angle $\psi = R + r$.
Therefore, centre distance for pressure angle β

$$= R_p + r_p = \frac{(R + r) \cos \psi}{\text{Cos } \beta}$$

where R = Known radius of wheel.
r = Known radius of pinion.
T = Circular thickness of tooth at radius R (in diametral plane).

t = Circular thickness of tooth at radius r (in diametral plane).
ψ = Known pressure angle at radii R and r (in diametral plane).
N = Number of teeth in wheel.
n = Number of teeth in pinion.
β = Pressure angle at meshing radii.
R_p = Meshing radius of wheel.
r_p = Meshing radius of pinion.

V.—SPIRAL GEARS.

Gears whose axes are non-parallel and non-intersecting are usually designated as spiral gears. Helical gears are a special application of the spiral gear, where the axes are parallel but non-intersecting. The shaft angle is determined from the helix angles and the helix hands of the mating gears. The hand of the helix is as equally important as the helix angle in determining the shaft angle, and for simplicity a right-hand helix or spiral may be regarded as positive and a left-hand may be regarded as negative. The shaft angle will then be the algebraic sum of the helix angles.

Thus, $\alpha_s = \alpha_w + \alpha_p$

where α_s = Shaft angle.
α_w = Helix angle of wheel at pitch point.
α_p = Helix angle of pinion at pitch point.

In the case of a pair of mating helical gears, the two helices are the same but of opposite hand. The shaft angle is therefore zero, *i.e.*, the axes are parallel.

Due to the helix angle, end thrust is set up, which may be computed in the same manner as for helical gears.

The helix angles need not necessarily be the same, but their algebraic sum must be equal to the shaft angle. Certain calculations of spiral gears must therefore be of a trial and error nature; this especially applies when the centre distance is fixed.

With gears, the axes of which are parallel and non-intersecting, the normal pitch and the circular pitch are the same for both mating gears. Mating spiral gears, the axes of which are at an angle, however, have the same normal pitch, but the circular pitch can vary with each gear. So that standard cutters may be used, a standard diametral pitch is chosen for the normal pitch of the gears, normal to the teeth.

$$\text{Circular pitch (diametral plane)} = \frac{\text{Circular pitch (normal plane)}}{\text{Cos spiral angle}}$$

Unless the spiral angles of both mating gears are the same, the velocity ratio cannot be determined from the pitch diameters. (Spiral gears are most efficient when the spiral angles are the same, and this condition should be satisfied whenever possible).

The velocity ratio is determined by the numbers of teeth in the wheel and pinion.

$$D = \frac{N}{p_n \cos a_w}$$

$$\text{and } d = \frac{n}{p_n \cos a_p}$$

$$\text{or } \operatorname{Cos} a_w = N/p_n D$$

$$\text{and } \operatorname{Cos} a_p = n/p_n d$$

$$\text{Centre distance} = \frac{D + d}{2}$$

$$= \frac{N}{2\, p_n \cos a_w} + \frac{n}{2\, p_n \cos a_p}$$

$$= \frac{1}{2\, p_n} \left(\frac{N}{\cos a_w} + \frac{n}{\cos a_p} \right)$$

$$\text{Therefore, } C = \frac{1}{2\, p_n} (N \text{ sec. } a_w + n \text{ sec. } a_p)$$

where N = Number of teeth in wheel.
n = Number of teeth in pinion.
a_w = Spiral angle wheel at pitch point.
a_p = Spiral angle of pinion at pitch point.
D = Pitch diameter of wheel.
d = Pitch diameter of pinion.
p_n = Diametral pitch (normal plane).
C = Centre distance.

If the centre distance, diametral pitch and numbers of teeth are fixed, the spiral angles may be found by trial and error.

Again, should the shaft angle, numbers of teeth and diametral pitch be fixed, and the centre distance unimportant, the spiral angles should be made equal and half the shaft angle, under which circumstances the gears are running at their greatest efficiency. When the spiral angles are equal, the velocity ratio is also the ratio of the two pitch diameters.

Individually, each spiral gear may be treated as a helical gear and tooth proportions, lead, base circle diameter, etc., computed accordingly.

Thus,

$$\text{Base circle diameter} = \frac{N \cos \psi_d}{p_n \cos a_w}$$

$$\text{and } \operatorname{Tan} \psi_d = \frac{\operatorname{Tan} \psi_n}{\operatorname{Cos} a}$$

where N = Number of teeth.
p_n = Diametral pitch (normal plane).
α_w = Spiral angle.
ψ_d = Pressure angle (diametral plane).
ψ_n = Pressure angle (normal plane).

Spiral Gears with Shafts at 90°

This is probably the most common case of spiral gears, the axes of which are non-parallel and non-intersecting. As a spiral angle cannot exceed 90°, the two mating spiral angles must be the same hand, either right or left, dependent on the direction of thrust required.

$$\text{Shaft angle} = 90^\circ$$

$$\text{Therefore,}\quad \alpha_w + \alpha_p = 90^\circ$$

$$\text{and}\quad \alpha_p = 90^\circ - \alpha_w$$

$$\text{Therefore,}\quad \operatorname{Cos}\alpha_p = \operatorname{Sin}\alpha_w$$

$$\text{Thus}\quad D = \frac{N}{p_n \cos\alpha_w} \quad\text{and}\quad d = \frac{n}{p_n \sin\alpha_w}$$

$$\text{and}\quad C = \frac{1}{2p_n}\left(\frac{N}{\operatorname{Cos}\alpha_w} + \frac{n}{\operatorname{Sin}\alpha_w}\right)$$

$$= \frac{1}{2p_n}(N \sec\alpha_w + n \operatorname{cosec}\alpha_w)$$

When the spiral angles are each 45°,

$$\cos\alpha_w = \sin\alpha_w$$

$$\text{and}\quad C = \frac{1}{2p_n}\left(\frac{N+n}{\operatorname{Cos}\alpha_w}\right)$$

$$= \frac{N+n}{2p_n\cos\alpha_w}$$

$$\text{and}\quad D/d = N/n$$

where N = Number of teeth in wheel.
n = Number of teeth in pinion.
α_w = Spiral angle of wheel.
α_p = Spiral angle of pinion.
D = Pitch dia. of wheel.
d = Pitch dia. of pinion.
p_n = Diametral pitch (normal plane).
C = Centre distance.

With the shafts at 90°, the circular pitch in the diametral plane of one spiral gear is equal to the axial pitch of the mating spiral gear, the axial pitch being the distance along the axis from one tooth to a similar point on the next tooth.

$$\text{Axial pitch of wheel} = \frac{p_n}{\text{Sin } \alpha_w}$$

$$\text{Circular pitch of pinion (diametral plane)} = \frac{p_n}{\text{Cos } \alpha_p}$$

and as $\sin \alpha_w = \cos \alpha_p$
Axial pitch of wheel = Circular pitch of pinion (diametral plane).

Contact between mating spiral gears (other than when the shafts are parallel, *i.e.*, helical gears) is only point contact, and this together with the high rubbing speed makes them suitable for light loads only. However, better contact can be obtained by using hollow-faced spiral gears, but these should be regarded rather as worm gears and designed accordingly.

Correction of Spiral Gears.

Spiral gears may be corrected to eliminate undercutting and interference in the same manner as that used for helical gears.

The "virtual" pitch dia. $= D/\cos^2 \alpha = D_v$
and the "virtual" number of teeth $= N/\cos^3 \alpha = N_v$

That is, the undercutting and interference to which a gear of N teeth will be subjected will be as for a spur gear of N_v teeth.

Theoretically, spiral gears whose axes are at 90° cannot run on extended centres without suitably correcting the spiral angles. Assuming the spiral angles to be α_w and α_p on the respective pitch diameters, each spiral angle would be slightly increased on the running pitch diameters when on extended centres. As the shaft angle with the same hand gears is equal to the sum of the mating spiral angles, the shaft angle would be slightly increased. With a drive of other than 90° and the mating gears of opposite hands, the algebraic sum would tend to cancel out the increase in spiral angles on the meshing diameters.

Due to the point contact of spiral gears, however, this slight discrepancy in shaft angle can usually be accommodated and correct tooth action maintained, but correct spiral angles are essential where hollow-faced spiral gears are used.

Gauging of Spiral Gear Teeth.

The tooth thickness may be checked by the gear tooth calliper as with helical gears, either checking the chordal thickness and corrected addendum or by the constant chord method.

VI.—STRAIGHT BEVEL GEARS.

Bevel gears are used where the axes of the gears are in the same plane and intersecting. The gear blanks are cones, therefore the teeth taper towards the vertex. The teeth are usually cut to a standard diametral pitch at the large end or back cone face.

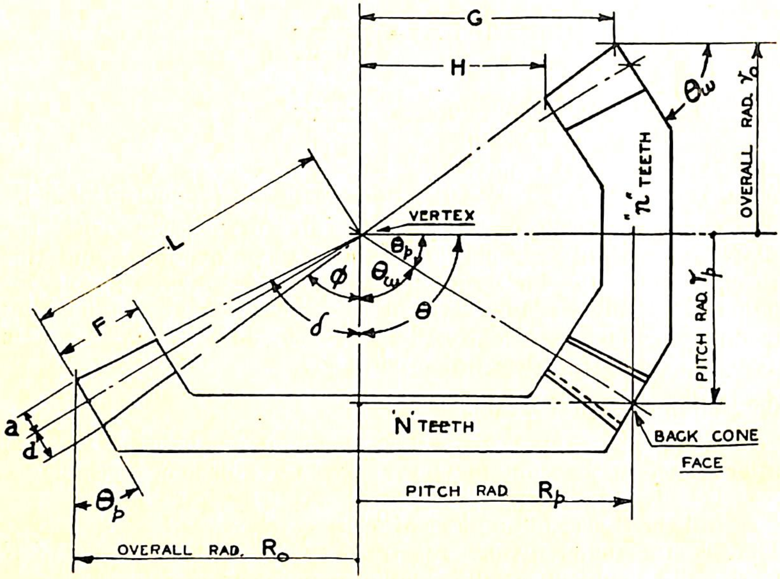


Fig. 29.

The velocity ratio is determined either by the number of teeth or the ratio of the pitch diameters (the two pitch cones rolling together without slip).

$$D_p/d_p = N/n = \text{Velocity ratio.}$$

Bevel Gears with Shafts at 90°.

Fig. 29 indicates a bevel gear drive at 90°

where R_p = Pitch radius of wheel.
R_0 = Outside radius of wheel.
r_p = Pitch radius of pinion.
r_0 = Outside radius of pinion.
θ = Shaft angle.
θ_w = Pitch cone angle of wheel.
θ_p = Pitch cone angle of pinion.
DP = Diametral pitch.
CP = Circular pitch.
ϕ_w = Root angle of wheel.
ϕ_p = Root angle of pinion.
δ_w = Face angle of wheel.
δ_p = Face angle of pinion.
L = Pitch cone distance.
a = Addendum.
d = Dedendum.
G_w = Vertex distance of wheel (large end).

G_p = Vertex distance of pinion (large end).
H_w = Vertex distance of wheel (small end).
H_p = Vertex distance of pinion (small end).

(1) $$R_p = \frac{\text{Number of teeth}}{2 \times DP} = \frac{N}{2\ DP} \text{ or } \frac{N \times CP}{2\ \pi}$$

also $$r_p = \frac{n}{2\ DP} \text{ or } \frac{n \times CP}{2\ \pi}$$

(2) Pitch cone angle of wheel θ_w $\quad \tan \theta_w = R_p/r_p$
also, $\quad \tan \theta_p = r_p/R_p$

The shaft angle is 90°
Therefore $\theta_w + \theta_p = \theta = 90°$

(3) Tan addendum angle $= a/L$
(4) Face angle $\delta_w = \theta_w +$ addendum angle.
also $\delta_p = \theta_p +$ addendum angle.
(5) Tan dedendum angle $= d/L$
(6) Root angle $\phi_w = \theta_w -$ dedendum angle.
also $\phi_p = \theta_p -$ dedendum angle.
(7) Pitch cone distance $L = R_p/\sin \theta_w$ or $r_p/\sin \theta_p$
(8) Outside radius $R_0 = R_p + a \sin \theta_p$
also $r_0 = r_p + a \sin \theta_w$
(9) Vertex distance $G_w = r_p - a \cos \theta_p$
also $G_p = R_p - a \cos \theta_w$

(10) Vertex distance $$H_w = \frac{G_w\ (L - F)}{L}$$

also $$H_p = \frac{G_p\ (L - F)}{L}$$

(11) Addendum at small end $$= \frac{a\ (L - F)}{L}$$

(12) Dedendum at small end $$= \frac{d\ (L - F)}{L}$$

(13) Circular pitch at small end $$= \frac{CP\ (L - F)}{L}$$

If in Nos. 10, 11, 12 and 13, F is taken as a distance from the large end, these formulae may be used to compute the tooth section at that distance.

Bevel Gears with Shafts at greater than 90°.

A bevel gear drive of this type is shown in Fig. 30.

(14) $AB = r_p$ = Pitch radius of pinion.
$AC = R_p$ = Pitch radius of wheel.

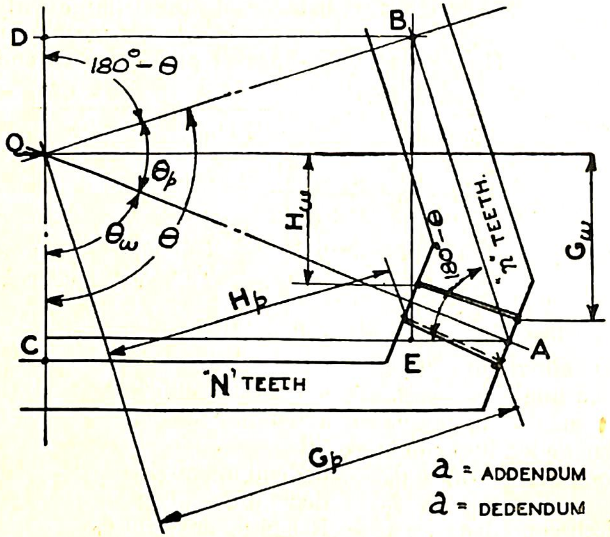


Fig. 30.

In triangle CAO, angle $C\hat{A}O = 90° - \theta_w$ and

in triangle OAB, angle $O\hat{A}B = 90° - \theta_p$

and as $\theta_w + \theta_p = \theta$, Angle $C\hat{A}B = 180° - \theta = B\hat{O}D$

$EA = r_p \cos (180° - \theta)$

Therefore $DB = R_p - r_p \cos (180° - \theta)$

$$OB = \frac{DB}{\mathrm{Sin}\ (180° - \theta)}$$

$$= \frac{R_p - r_p \cos (180° - \theta)}{\mathrm{Sin}\ (180° - \theta)}$$

$$\mathrm{Tan}\ \theta_p = \frac{r_p}{OB} = \frac{r_p \sin (180° - \theta)}{R_p - r_p \cos (180_0 - \theta)}$$

Divide by r_p

$$\mathrm{Tan}\ \theta_p = \frac{\mathrm{Sin}\ (180° - \theta)}{(R_p/r_p) - \cos (180° - \theta)}$$

$$\text{or}\quad \mathrm{Tan}\ \theta_p = \frac{\mathrm{Sin}\ (180° - \theta)}{(N/n) - \cos (180° - \theta)}$$

and $\theta_w = \theta - \theta_p$

(15) Vertex distance of wheel (large end) G_w

$$= \frac{R_p}{\operatorname{Tan} \theta_w} - a \cos (90^\circ - \theta_w)$$

$$= \frac{R_p}{\operatorname{Tan} \theta_w} - a \sin \theta_w$$

$$\text{also } G_p = OB - a \cos (90^\circ - \theta_p)$$

$$= OB - a \sin \theta_p$$

$$= \frac{R_p - r_p \cos (180^\circ - \theta)}{\operatorname{Sin} (180^\circ - \theta)} - a \sin \theta_p$$

Other calculations required are as for a bevel drive at 90°

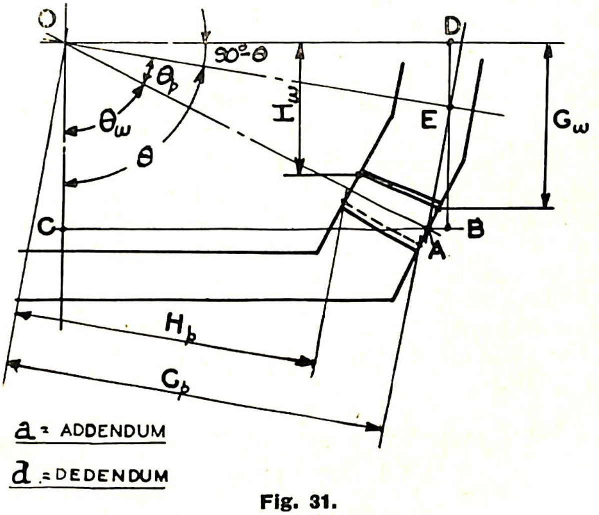


Fig. 31.

Bevel Gears with Shafts at less than 90°

A bevel gear drive of this type is shown in Fig. 31.

(16) $AE = r_p$ = Pitch radius of pinion.
$AC = R_p$ = Pitch radius of gear.

$$\text{Angle } E\hat{A}B = \theta$$

$$AB = r_p \cos \theta$$

$$CB = CA + AB = R_p + r_p \cos \theta = OD$$

$$OE = \frac{OD}{\operatorname{Cos} (90^\circ - \theta)} = \frac{OD}{\operatorname{Sin} \theta} = \frac{R_p + r_p \cos \theta}{\operatorname{Sin} \theta}$$

$$\operatorname{Tan} \theta_p = \frac{r_p}{OE} = \frac{r_p \sin \theta}{R_p + r_p \cos \theta}$$

Divide by r_p

$$\text{Tan } \theta_p = \frac{\text{Sin } \theta}{(R_p/r_p) + \cos \theta}$$

$$\text{or, Tan } \theta_p = \frac{\text{Sin } \theta}{(N/n) + \cos \theta}$$

(17) Vertex distance of wheel G_w

$$= \frac{R_p}{\text{Tan } \theta_w} - a \sin \theta_w$$

$$\text{also, } G_p = OE - a \cos (90° - \theta_p)$$

$$= OE - a \sin \theta_p$$

$$= \frac{R_p + r_p \cos \theta}{\text{Sin } \theta} - a \sin \theta_p$$

Other calculations required are as for a bevel drive at 90°.

Correction of Straight Bevel Gears.

The tooth shape and proportions of a bevel gear at N teeth are as for a straight spur gear having a number of teeth equal to the "virtual number of teeth" of the bevel gear. Reference to Fig. 32 shows the development of the gear teeth on the back cone face. The tooth shape at the large end of the pinion of pitch radius r_p would be as for a spur gear of pitch radius QP. Likewise, the tooth shape of the wheel of pitch radius R_p would be as for a spur gear of pitch radius OP.

$$\text{Virtual pitch radius of pinion} = QP$$

$$= \frac{\text{Pitch radius } r_p}{\text{Cos } \theta_p}$$

Number of teeth = 2 pitch radius × diametral pitch.
Thus virtual number of teeth of pinion

$$= \frac{\text{Number of teeth in pinion}}{\text{Cos } \theta_p}$$

Also virtual number of teeth in wheel

$$= \frac{\text{Number of teeth in wheel}}{\text{Cos } \theta_w}$$

The pressure angle of a bevel gear is taken on the back cone face.

Hence base circle radius

$$= \text{Virtual pitch radius} \times \text{Cos } \psi$$

where ψ = Pressure angle.

Bevel gears of a small number of teeth are subjected to interference and undercutting in the same manner as straight spur gears. Fewer teeth, however, may be employed before this takes place, as undercutting and interference are dependent on the virtual number of teeth and not the actual. Bevel gear teeth may be corrected to avoid this condition and the same methods adopted as those used for straight spur gears. The gears are treated as spur gears of the virtual number of teeth developed on the back cone face, as shown in Fig. 32.

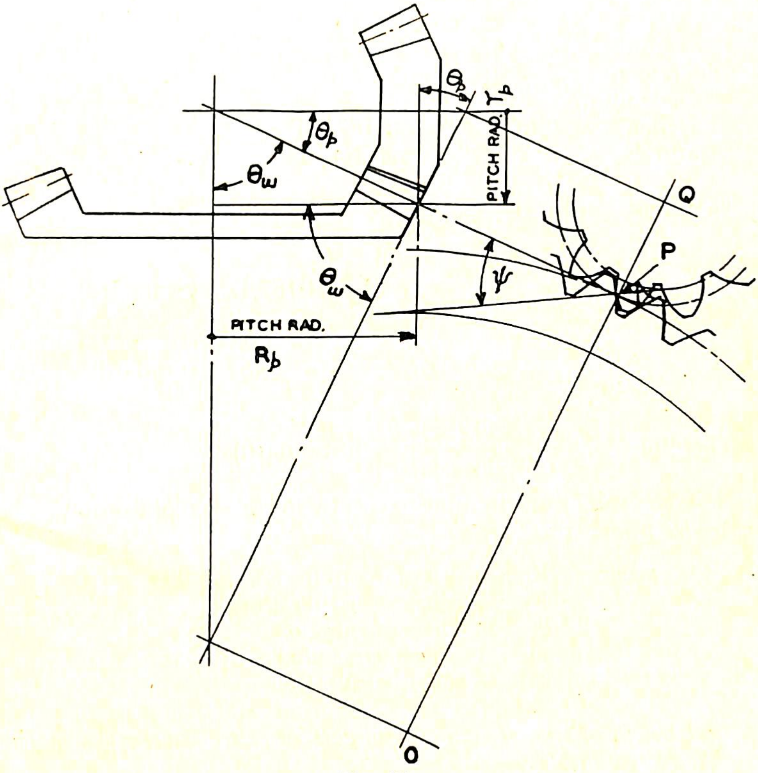


Fig. 32.

Tooth Action of Bevel Gears.

It is essential that there be continuity of tooth action with bevel gears as with other types of gearing. For studying the tooth action of bevel gears, the plane of the back cone face is taken, as in this plane a true view of the teeth profiles is obtained. The gears may then be treated as straight spurs, and the formulae for straight spurs used, but the virtual gear diameters taken and not the actual diameters. In a number of instances only the formulae are given as the development of the formulae is given in the similar applications for the straight spur gear.

As with straight spur gears, for the involute not to be destroyed above the base circle diameter by the tip of the cutter, contact between the cutter and the gear being cut must always be on the line of action.

Referring to Fig. 10, R_r is therefore the minimum root radius of the gear for the involute not to be destroyed above the base circle diameter.

$$R_r = R_b \cos \psi$$

$$\text{But} \quad R_b = R_p \cos \psi$$

$$\text{Therefore} \quad R_r = R_p \cos^2 \psi$$

$$\text{Dedendum} = R_p\,(1 - \cos^2 \psi)$$

$$\text{Dedendum (Standard)} = 1{\cdot}157/DP.$$

$$\text{and } R_p = N_v/2\,DP \qquad \text{and } N_v = N/\text{Cos } \theta_w.$$

$$\text{Therefore} \quad R_p = \frac{N}{2\,DP \cos \theta_w}$$

$$\text{Thus} \quad \frac{N}{2\,DP \cos \theta_w}\,(1 - \cos^2 \psi) = 1{\cdot}157/DP \text{ (Standard)}$$

$$\frac{N}{2 \cos \theta_w}\,(1 - \cos^2 \psi) = 1{\cdot}157 \text{ (Standard)}$$

$$\text{Therefore} \quad N = \frac{2{\cdot}314 \cos \theta_w}{1 - \cos^2 \psi} \text{ (Standard)}$$

N is the minimum number of teeth for the involute not to be destroyed above the base circle.

where R_p = Virtual pitch radius.
R_b = Virtual base radius.
ψ = Pressure angle.
θ_w = Pitch cone angle of gear.
DP = Diametral pitch.

To find length of line of contact and contact ratio between two mating straight bevel gears.

Referring to Fig. 11,

Length of line of contact

$$= \sqrt{r_0^2 - r_b^2} + \sqrt{R_0^2 - R_b^2} - (r_b + R_b) \tan \psi$$

Contact ratio

$$= \frac{\sqrt{r_0^2 - r_b^2} + \sqrt{R_0^2 - R_b^2} - (r_b + R_b) \tan \psi}{\text{Circular pitch} \times \cos \psi}$$

where R_0 = Virtual outside radius of wheel.
R_b = Virtual base radius of wheel.
r_0 = Virtual outside radius of pinion.
r_b = Virtual base radius of pinion.
ψ = Pressure angle.

To find arcs of approach and recession of two mating straight bevel gears.

Referring to Fig. 11. Assuming gear A to be the driver.

Arc of approach of gear A

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{\cos \psi}$$

Arc of approach of gear A (radians)

$$= \frac{\sqrt{R_0^2 - R_b^2} - R_b \tan \psi}{r_b}$$

Arc of recession of gear A

$$\frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{\cos \psi}$$

Arc of recession of gear A (radians)

$$\frac{\sqrt{r_0^2 - r_b^2} - r_b \tan \psi}{r_b}$$

where R_0 = Virtual outside radius of wheel.
R_b = Virtual base radius of wheel.
r_0 = Virtual outside radius of pinion (gear A).
r_b = Virtual base radius of pinion (gear A).
ψ = Pressure angle.

Gauging of Straight Bevel Gear Teeth.

Bevel gear tooth thickness may best be gauged and checked by the tooth calliper. Due to the tooth profile tapering towards the vertex, the teeth must either be checked at the large end or at some definite position along the tooth face. The following formulae are based on the assumption that the teeth are gauged at the large end, but they are still applicable for gauging at any other point providing the virtual diameter and tooth proportions are taken at that point.

Referring to Fig. 18.

To find the Chordal Thickness.

For standard gears, the circular thickness at the pitch diameter, ignoring backlash.

$$= \frac{D_v \times \pi}{2 \times N_v} = \frac{\text{Circular pitch}}{2}$$

For corrected gear teeth, compute the circular thickness at the virtual pitch diameter. See "Correction of Straight Spur Gears."

$$\text{Then } \theta \text{ (degrees)} = \frac{360 \times \text{Circular thickness at } D_v}{D_v \times \pi \times 2}$$

$$\text{Chordal thickness} = 2 \times R_v \times \sin \theta$$

where D_v = Virtual pitch diameter.
N_v = Virtual number of teeth.
R_v = Virtual pitch radius.

To find Corrected Addendum H.

$$h = R_v \ (1 - \text{Cos } \theta)$$

Corrected addendum H = h + addendum.

Constant Chord Calliper Setting.

Referring to Fig. 19.

Constant chord thickness

$$= \frac{\text{Circular pitch} \times \cos^2 \psi}{2}$$

Constant chord height

$$= \text{Addendum} - \frac{\text{Circular pitch} \times \sin \psi \cos \psi}{4}$$

It also follows that there is a "Constant Chord" setting for all gears corrected by the same amount and cut by the same cutter.

APPENDIX I.

When the angle is known, the involute function may be accurately obtained from

$$\text{Inv. } \psi = \tan \psi - \psi \text{ (radians)}$$

When, however, the value of the involute function is given and the corresponding angle is required, the following table may prove useful, but as only every 15′ up to 45° is given, its limitations are fully realised. In the absence of a full set of tables, if the approximate angle for a given value of the involute function is known, the true angle may be obtained by trial and error.

°	0′	15′	30′	45′
1	·00000	·00000	·00001	·00001
2	·00001	·00002	·00003	·00004
3	·00005	·00006	·00008	·00009
4	·00011	·00014	·00016	·00019
5	·00022	·00026	·00029	·00034
6	·00038	·00044	·00049	·00055
7	·00061	·00068	·00075	·00083
8	·00091	·00100	·00110	·00120
9	·00130	·00142	·00154	·00166
10	·00179	·00193	·00208	·00223
11	·00239	·00256	·00274	·00292
12	·00312	·00332	·00353	·00375
13	·00397	·00421	·00446	·00472
14	·00498	·00526	·00554	·00584
15	·00615	·00647	·00680	·00714
16	·00749	·00786	·00823	·00862
17	·00902	·00944	·00986	·01031
18	·01076	·01123	·01171	·01221
19	·01272	·01324	·01378	·01433
20	·01490	·01549	·01609	·01671
21	·01734	·01800	·01866	·01935
22	·02005	·02077	·02151	·02227
23	·02305	·02384	·02466	·02549
24	·02635	·02722	·02812	·02904
25	·02998	·03093	·03192	·03292
26	·03395	·03500	·03607	·03717
27	·03829	·03943	·04060	·04180
28	·04302	·04426	·04553	·04684
29	·04816	·04952	·05090	·05231
30	·05375	·05522	·05672	·05825
31	·05981	·06140	·06302	·06468
32	·06636	·06808	·06984	·07163
33	·07345	·07531	·07720	·07913
34	·08110	·08310	·08514	·08722
35	·08934	·09150	·09370	·09594
36	·09822	·10055	·10291	·10533
37	·10778	·11028	·11283	·11542
38	·11806	·12075	·12348	·12627
39	·12911	·13199	·13493	·13792
40	·14097	·14407	·14722	·15043
41	·15370	·15703	·16041	·16386
42	·16737	·17093	·17457	·17826
43	·18202	·18585	·18975	·19371
44	·19774	·20185	·20603	·21028
45	·21460	·21900	·22348	·22804

APPENDIX II.

Tooth Proportions for Diametral Pitch Gears.

Diametral Pitch.	Circular Thickness.	Addendum.	Dedendum (Standard).	Dedendum (Fellows).
4	·3927	·2500	·2893	·3125
5	·3142	·2000	·2314	·2500
6	·2618	·1666	·1928	·2083
7	·2244	·1429	·1653	·1786
8	·1964	·1250	·1446	·1563
9	·1745	·1111	·1286	·1389
10	·1571	·1000	·1157	·1250
11	·1428	·0909	·1052	·1136
12	·1309	·0833	·0964	·1042
14	·1122	·0714	·0826	·0893
16	·0982	·0625	·0723	·0781
18	·0873	·0555	·0643	·0705
20	·0785	·0500	·0579	·0650
22	·0714	·0455	·0526	·0604
24	·0654	·0417	·0482	·0566
26	·0604	·0385	·0445	·0533
28	·0561	·0357	·0413	·0507
30	·0524	·0333	·0386	·0483
32	·0491	·0312	·0362	·0462
34	·0462	·0294	·0340	·0394
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5. Horse-Power of Leather Belts, etc.
6. Automobile Brakes (Axle Brakes)
7. Automobile Brakes (Transmission Brakes) } Connected.
8. Capacities of Bucket Elevators.
9. Valley Angle Chart for Hoppers and Chutes.
10. Shafts up to 5½-in. diameter, subjected to Twisting and Combined Bending and Twisting,
11. Shafts, 5¾ to 26-inch diameter, subjected to Twisting and Combined Bending and Twisting.
12 Ship Derrick Booms.
13. Spiral Springs (Diameter of Rd. or Sq. Wire)
14. Spiral Springs (Compression).
15. Automobile Clutches (Cone Clutches).
16. ,, ,, (Plate Clutches).
17. Coil Friction for Belts, etc.
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19. Internal Expanding Brakes. Angular Proportions for Self-Balancing } Connected.
20. Referred Mean Pressure Cut-Off, etc.
21 Particulars for Balata Belt Drives.
22. ⅞" Square Duralumin Tubes as Struts
23. 1" ,, ,, ,, ,,
24. ¾" Sq. Steel Tubes as Struts 30 ton yield).
25. ⅞" ,, ,, ,, (30 ton yield).
26. 1" ,, ,, ,, (30 ton yield).
27. ¾" ,, ,, ,, (40 ton yield).
28. ⅞" ,, ,, ,, (40 ton yield).
29. 1" ,, ,, ,, (40 ton yield).
30. Moments of Inertia of Built-up Sections (Tables)
31. Moments of Inertia of Built-up Sections (Instructions and Examples) } Connected.
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33. Reinforced Concrete Slabs (Instructions and Examples) } Connected.
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37. ,, ,, ,, (Sheet 3, Notes and Examples)
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42. Useful Conversion Factors.
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46. Coned Plate Development.
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49. Relation between Length, Linear Movement and Angular Movement of Lever. (Diagram and Notes)
50. ,, ,, ,, ,, ,, ,, (Chart).

51. Helix Angle and Efficiency of Screws and Worms.
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55. ,, ,, ,, (Square Wire)
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